Weighted mean aerosol scattering phase function

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Models of scattering phase functions of polydisperse aerosol are presented for $\lambda = 0.4$, 0.55, and 0.8 μm and weighted mean microstructure characteristics related to the atmospheric aerosol column.

When performing the atmospheric optics calculations, the necessity often appears to have data on the weighted mean scattering phase function related to the atmospheric aerosol column. In particular, such data are necessary for:

– calculation of the mean (standard) values of upwelling and downwelling fluxes of scattered shortwave radiation, atmospheric transmission, planetary albedo of the atmosphere and the "Earth – atmosphere" system;

– atmospheric correction of the results of optical measurements when performing geological, hydrological, forest engineering and other investigations of the Earth from space;

– lighting engineering calculations related to radiation scattered within different parts of the sky.

The usefulness of creating handbook models of the weighted mean aerosol scattering phase functions related to the atmospheric aerosol column for certain landscape–climatic conditions are doubtless, for example, for continental and marine aerosols in different seasons, at different heights, in different wavelength ranges, and so on.

However, such models have not yet been described in literature, because there are some serious methodological difficulties in the development of such models. There are no data of direct measurements of the aforementioned scattering phase functions, because it is not clear yet, how to perform them.

Some researchers undertook attempts to reconstruct such scattering phase functions from the measurement data on the brightness phase functions of the daytime clear sky. These attempts were mainly of the research character, and have used a number of simplifying assumptions and suppositions; hence, they were not characterized by high accuracy and reliability. It is not possible to determine the statistically provided weighted mean scattering phase functions from thus obtained measurement data.

We propose to use the model scattering phase functions calculated by the Mie formulas for the atmospheric aerosol column at the weighted mean characteristics of its microstructure as the weighted mean aerosol scattering phase functions in different wavelength ranges. Undoubtedly, such an approach is approximate from the statistical and mathematical point of view. However, no ways are seen to more accurately solve this problem in the nearest future.

Based on the approach formulated above, let us reduce the problem to the determination of the shape and parameters of the weighted mean particle size distribution related to the atmospheric aerosol column as well as the weighted mean value of the complex refractive index $m = n - \imath\, i$.

However, even in that simplified form solution of this problem seems to be highly problematic, because there are no data on the microstructure characteristics of aerosol in the atmospheric column.

It is the encouraging fact that there are relatively long series of measurement data on the optical and microphysical aerosol characteristics at different heights, in different wavelength ranges, under different climatic conditions, and so on. Using these data as well as the available model representations, we tried, wherever it was possible, to approximately estimate the weighted mean characteristics of the aerosol microstructure in the atmospheric column. These characteristics were determined within the frameworks of the aerosol model, which assumes the particles to be spherical, homogeneous and having the same complex refractive index of the particulate matter for particles of any size.

The reasons for accepting such a simulation are in the fact that the peculiarities in the microstructure of individual aerosol ensembles are smoothed and disappear at statistical averaging of large arrays of measurement data.

The shape of the weighted mean size distribution function

In the studies presented in Refs. 1 and 2 the shape and parameters of the weighted mean aerosol particle-size distribution functions were determined. The phenomenological histogram of the particle size distribution was obtained by averaging more than 250 different most representative measurements of the aerosol particle size distributions. This histogram can be

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well approximated for the optically active particles with the radii $r = 0.01–1.0$ μm by the following formula:

$$f^*(r) = \frac{dN}{d \log r} = \frac{A}{r_0^{\bar{\nu}} + | r - r_0 |^{\bar{\nu}}}, \qquad (1)$$

where r_0 is the modal radius of the particle size spectrum and $\bar{\nu}$ is the disperse index of aerosol particles.

Then, samples were grouped, from the entire array of measurements, that represent the continental and marine, tropospheric and stratospheric aerosol. The weighted mean histograms of the size distribution were obtained by averaging over each sample.

As analysis has shown, all of them can be approximated by the function of the form (1) in the particle size range $r_0/3 \le r \le 1$ μm, and different values of the distribution parameters r_0 and $\bar{\nu}$.

Thus, it was experimentally obtained that the form of the particle size distribution function is invariant relative to the aerosol of different origin, with only the distribution parameters r_0 and $\bar{\nu}$ being different for different aerosols.

Based on this fact, the approximation function (1) was accepted for calculation of the weighted mean aerosol size distribution function.

Weighted mean values of the distribution parameters

As it follows from analysis,[1,2] the weighted mean values of the parameters of tropospheric aerosol particle size distribution are $r_0 = 0.03$ μm and $\bar{\nu} = 3.0$.

However, one should have in mind that these values were obtained from the measurement data on particle size distributions in the near-ground layer. There were no sufficient reasons to extend these values to the atmospheric aerosol column.

The fact favors the possibility of obtaining the statistically provided value of the disperse index $\bar{\nu}$ related to the atmospheric column that this index is closely related to the index of spectral behavior (Angström parameter ω_a) of the optical thickness of the atmospheric aerosol τ_a.

At the inverse-power distribution of particles

$$dN/(d \log r) \sim r^{-\nu^*}$$

and sufficiently wide size spectrum

$$\nu^* = \omega_a + 2. \qquad (2)$$

As our calculations have shown, the indices $\bar{\nu}$ and ω_a at the weighted mean distribution of particles (1) and $r_0 = 0.03$ μm are related to each other by the approximate relationship

$$\bar{\nu} = 7(\omega_a + 2)/8. \qquad (3)$$

The Angström parameter can be determined from the data of measuring the aerosol optical thickness $\tau_a(\lambda)$ at two wavelengths λ_1 and λ_2 by the formula

$$\omega_a = \ln \frac{\tau_a(\lambda_1)}{\tau_a(\lambda_2)} / \ln \frac{\lambda_2}{\lambda_1}.$$

The spectrum of aerosol optical thickness $\tau_a(\lambda)$ is regularly measured at the International Network of Actinometric and Ozonometric Stations of Hydrometeorological Service and at the astronomy observatories throughout the world. The series of long-term measurements of the Angström index ω_a have been compiled to date. The results of generalization of measurements of the Angström index are presented below. The data were obtained by the following investigators: A. Angstrom (1964) $\omega_a = 1.3$; F.E. Foltz (1956) – 1.2; G.P. Gushchin and T.A. Pavlyuchenkova (1963) – 1.32; I. Wempe (1947) – 1.33; E.A. Polyakova (1976) – 1.3; G.P. Gushchin (1988) – 1.0; O.D. Barteneva, N.I. Nikitinskaya, G.G. Sakunov, and L.K. Veselova (1991) – 1.3. As is seen, the statistical data of the majority of investigators, except the data by G.P. Gushchin (1988) are close to each other and are, in average, equal to $\bar{\omega}_a = 1.28$.

Therefore, according to Eq. (3), the value $\bar{\nu} = 2.88$ can be accepted as the weighted mean disperse index related to the atmospheric aerosol column.

It was mentioned above that the approximation function (1) was obtained for the optically active particles with the size $r = 0.01–1.0$ μm. Sedimentation of particles affects the particles with the radii $r > 1$ μm and, then the number density decreases more quickly than it follows from Eq. (1). The replacement of the disperse index $\bar{\nu}$ in Eq. (1) for $\tilde{\nu} = \bar{\nu} + 0.1\, r$ (r is in μm) (Ref. 3) makes it possible to take into account the effect of sedimentation of large particles. Thus, the weighted mean particle-size distribution function at $r \ge r_0/3$ is the following:

$$f^*(r) = \frac{dN}{d \log r} = \frac{A}{r_0^{\bar{\nu}} + | r - r_0 |^{\tilde{\nu}}}, \qquad (4)$$

where $\tilde{\nu} = 2.88 + 0.1\, r$ (r is in μm).

Figure 1 illustrates the possibilities of approximating the empirically obtained particle size distributions by the function of the form (4) by means of selecting the parameters r_0, $\bar{\nu}$, and A. The figure shows the mean values of the particle distribution functions of continental (curve *1*) and marine (curve *2*) aerosols obtained by G. Hanel and K. Bullrich[4] as well as the values of the function (4) at approximation of the particle size distribution of continental aerosol ($\bar{\nu} = 3.183$ (---); $r_0 = 0.05$ μm; $A = 1.424 \cdot 10^{-4}$) – curve *1′* and marine aerosol ($\bar{\nu} = 2.727$ (---); $r_0 = 0.12$ μm; $A = 5.1 \cdot 10^{-3}$) – curve *2′*. The values of the function (1)

without the account of sedimentation of large particles are shown by dot–and–dash line.

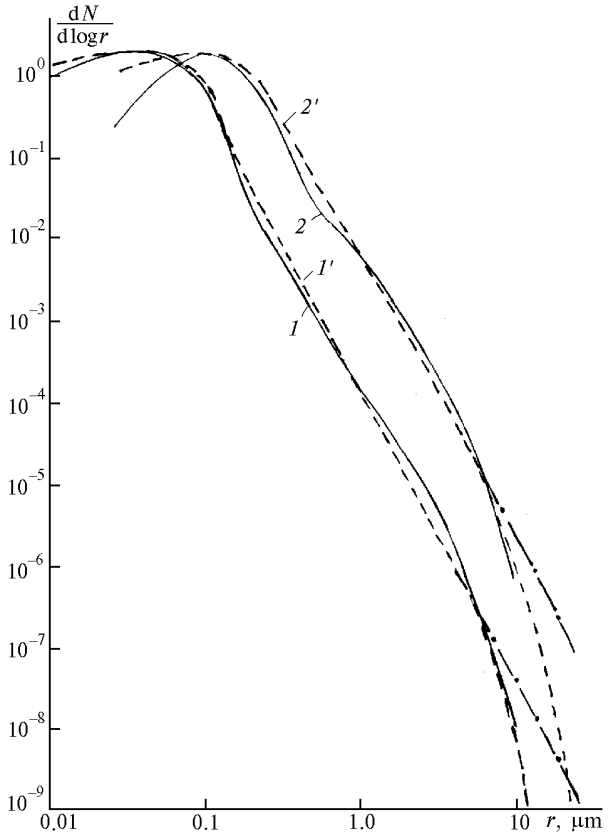

Fig. 1. Aerosol particle-size distribution for continental aerosol[4] (*1*) and marine aerosol[4] (*2*) and approximation of the size distribution of continental (*1′*) and marine (*2′*) aerosol, respectively, by the function (4). Dot–and–dash curves show the approximation by the function (1).

It is seen from this figure that:

– empiric curves of the particle size distribution obtained by averaging over a significant number of measurements are quite well approximated by the function of the form (4) at the corresponding values of the parameters r_0, $\bar{\nu}$ and A;

– the particle size distributions approximated by Eq. (4) are in better agreement with the measured ones if sedimentation of particles is taken into account.

It is also difficult to experimentally determine the modal size r_0 related to the column atmospheric aerosol because only the data of some investigators on the values r_0 at some heights are available. Generalization of the literature data shows that the values r_0 under different conditions can vary within a wide range from 0.0015 to 0.15 (sometimes up to 0.2) μm. Modal radii of particles in the lower atmosphere (near-ground and boundary layers) are usually 0.02–0.05 μm. The values r_0 in the upper troposphere (h = 7–12 km) decrease to 0.01–0.035 μm. The large values r_0 = 0.1–0.2 μm can be observed in the lower stratosphere, especially in the sulfate Junge layer (h = 16–20 km).

On the whole, the estimates made in Ref. 2 have shown that the value r_0 = 0.03 μm can be accepted as the mean value r_0 in the troposphere. According to the estimates,[3] the weighted mean values related to the atmospheric aerosol column are the same.

Weighted mean value of the complex refractive index of particles

One can determine the weighted mean value of the complex refractive index of particles if the weighted mean values of the refractive index n and the absorption index ι are available.

The refractive index of natural particulate matter can have the values from n = 1.38 to n = 1.65. The refractive index values n from 1.43 to 1.53 are observed most often (see, for example, Table 5 in Ref. 5). The generally accepted value n is n = 1.5. The majority of polydisperse scattering phase functions calculated by researchers from Russia and other countries for different particle-size distribution functions used just this refractive index.

However, it is methodologically more correct to calculate the weighted mean scattering phase functions by averaging the scattering phase functions obtained at different values of the refractive index. In so doing we have calculated the scattering phase functions for the weighted mean particle size distribution and the refractive index values n = 1.43, 1.50 and 1.53 by the spectral–zonal method. The weighted mean values of the scattering phase functions were calculated taking into account the weighting functions $P(n)$ of the refractive index n close to the values 1.43, 1.50 and 1.53:

$$\tilde{\gamma}_a(\theta) = \sum_{i=1}^{3} \gamma_a(\theta, n_i)\, P(n_i).$$

The following values of the weighting functions $P(n_i)$ were accepted in calculations:

$$P(1.43) = 0.15;\ P(1.5) = 0.55;\ P(1.53) = 0.30.$$

The weighted mean values of $\tilde{n}$ were determined by the following formula:

$$\tilde{n} = \sum_{i=1}^{3} n_i\, P(n_i).$$

The value $\tilde{n}$ found using the accepted values of the weighting functions is $\tilde{n}$ = 1.4985, i.e., it practically coincides with the generally accepted value $\bar{n}$ = 1.5.

It is also very problematic to determine the weighted mean value of the absorption index related to the atmospheric aerosol column. There are only few data available from literature on the absorption indices of the particles of different origin and different size,

and they are very contradictory. It is only during recent years that investigators have begun to draw some conclusions that natural aerosol absorbs the radiation very weakly. E. Patterson[6] notes that even volcanic ashes have the absorption index $\iota \le 0.005$.

According to data from Refs. 7 and 8 the absorption index of "dry matter" of natural aerosol in the visible range does not leave the interval 0.005–0.007 in the majority of cases. As it seems to us the size and mass of particles increase because of moistening of particles at the presence of water vapor in the atmosphere, that leads to a decrease of the absorption index even at the same quantity of the absorbing substance. The data obtained by M.V. Kabanov, M.V. Panchenko, Yu.A. Pkhalagov, et al.[9] confirm that actual values of the absorption index in the atmosphere, *in situ,* do not exceed $\iota = 0.002$.

We calculated the scattering phase functions by the spectral–zonal aerosol scattering functions[3] tabulated only for discrete values of the absorption index $\iota = 0$, 0.005, 0.02, 0.035 and 0.05. So the weighted mean scattering phase functions for the value $\iota = 0.002$ were calculated by the interpolation formula

$$\tilde{\gamma}_a(\theta) = \gamma_a(\theta, \iota = 0)\, P(\iota = 0) + \\ + \gamma_a(\theta, \iota = 0.005)\, P(\iota = 0.005)$$

at the values of the coefficients $P(\iota = 0) = 0.6$ and $P(\iota = 0.005) = 0.4$.

Such an approach to determination of the weighted mean aerosol scattering phase function is methodologically more correct than calculation directly for $\iota = 0.002$, because it makes it possible to take into account the realizations of the scattering phase functions in the interval ι from 0 to 0.005.

The weighted mean value of the absorption index ι determined by the formula

$$\tilde{\iota} = \sum_{i=1}^{2} \iota_i\, P(\iota_i), \text{ and is equal to } 0.002.$$

Models of the weighted mean scattering phase functions for the wavelength 0.4, 0.55 and 0.8 μm

Taking into account the aforementioned derivations, the weighted mean aerosol scattering phase functions were calculated by the formula

$$\tilde{\gamma}_a(\theta) = [0.09\, \gamma(\theta; 1.43) + 0.33\, \gamma(\theta; 1.50) + \\ + 0.18\, \gamma(\theta; 1.53)]_{\iota = 0} + [0.06\, \gamma(\theta; 1.43) + \\ + 0.22\, \gamma(\theta; 1.50) + 0.12\, \gamma(\theta; 1.53)]_{\iota = 0.005}. \qquad (5)$$

Calculations of the function $\tilde{\gamma}_a(\theta)$ were carried out for the wavelengths $\lambda = 0.4$, 0.55 and 0.8 μm. The polydisperse scattering phase functions $\gamma_a(\theta, m)$ included into Eq. (5) were calculated by the spectral–zonal method for aerosol with the mean size distribution weighted by Eq. (4) in the size range from 0.0127 to 26 μm at the values of the distribution parameters $\bar{\nu} = 2.88$ and $r_0 = 0.03$ μm. The results of calculations are presented in Table 1.

Table 1. Weighted mean aerosol scattering phase functions

Scattering angle θ°	Wavelength, μm		
	0.4	0.55	0.8
0	208	169	131
1	49.0	47.5	45.0
2	29.1	28.6	27.9
4	19.0	19.2	19.8
6	14.5	14.7	15.2
8	11.5	11.7	12.0
10	9.62	9.85	10.1
15	7.05	7.14	7.24
20	5.44	5.46	5.49
30	3.34	3.37	3.34
40	2.16	2.13	2.09
50	1.39	1.36	1.34
60	0.908	0.892	0.875
70	0.610	0.601	0.593
80	0.427	0.424	0.420
90	0.318	0.317	0.316
100	0.252	0.253	0.255
110	0.215	0.218	0.222
120	0.199	0.204	0.210
130	0.199	0.206	0.215
140	0.215	0.224	0.236
150	0.251	0.264	0.278
160	0.309	0.325	0.342
170	0.351	0.367	0.387
180	0.428	0.450	0.475
Γ_a	7.38	7.17	6.94

It is interesting to compare the obtained scattering phase functions $\tilde{\gamma}_a(\theta, \lambda)$ with the more known scattering phase functions obtained by different authors for the atmospheric aerosol column.

First of all, they are the scattering phase functions determined from the measurement data on brightness phase functions of the daytime clear sky. The majority of them (see, for example, Refs. 10–15) were obtained by the simplest semi-empiric relationships accurate to about 30%.[12,13] Three aerosol scattering phase functions determined by such semi-empiric relationships from the data presented in Refs. 10, 11, 14, and 15 are presented in Table 2. The aerosol scattering phase functions presented in Refs. 14, 15, and 16 were obtained in Ref. 14 from the data of Refs. 15 and 16 by excluding the scattering phase functions of the molecular components.

Table 2. Normalized aerosol scattering phase functions from Refs. 10, 11, 14–19

Ref.	10	11	14, 15	14, 16	17	18	19
θ°	θ°/λ, μm						
	0.546	0.700	0.543	0.4–0.7	0.547	0.55	0.53
0	324*	880*	240*	190*	1005	79.4	606
1	215*	260*	82.0*	42.0*	506*	60.2	331
2	111	123	48.7	29.0*	183	37.0	132*
4	39.5	39.5*	23.9	18.7*	56.4	21.7	39.8*
6	21.1	21.1	16.3	14.9*	25.9	16.0	21.6*
8	13.8	13.8*	12.9	12.0*	15.6	12.9	14.5*
10	10.3	11.7	10.8	10.5	10.8	10.8	11.1
15	6.45	6.29	7.31	8.20*	5.92	7.50	7.38
20	4.40	4.60	5.58	6.18	4.10	5.74	5.25*
30	2.63	2.71	3.60*	3.65*	2.54	3.41	3.11
40	1.68	1.82	2.22	2.15	1.58	2.08	1.84*
50	1.12	1.20*	1.37*	1.33*	1.03	1.30	1.07*
60	0.756	0.829	0.873	0.855	0.700	0.830	0.630*
70	0.535*	0.560	0.560*	0.560*	0.474	0.551	0.406
80	0.398	0.402	0.380	0.415	0.338	0.382	0.295*
90	0.313	0.289*	0.270	0.338*	0.256	0.279	0.212
100	0.278	0.251	0.194	0.289	0.206	0.218	0.135
110	0.252*	0.216	0.160*	0.255*	0.178	0.185	0.124
120	0.250	0.201	0.147	0.239	0.168	0.172	0.112
130	0.264*	0.205*	0.145*	0.235*	0.168	0.175	0.113
140	0.280	0.226	0.152	0.235	0.185	0.195	0.145*
150	0.320*	0.250*	0.154*	0.240*	0.211	0.244	0.199
160	0.382	0.276	0.157	0.245	0.234	0.336	0.229
170	0.430	0.315*	0.190*	0.250*	0.264	0.338	0.273
180	0.450*	0.355*	0.260	0.255	0.312	0.485	0.352
Γ_a	5.92	7.75	10.8	7.08	9.00	8.17	12.3

* The values $\gamma_a(\theta, \lambda)$ are obtained by extrapolation and interpolation methods.

The more reliable inversion of the measurement data on the sky brightness to the aerosol scattering phase functions can be obtained by inverting the radiation transfer equation. Such an inversion was first made in Ref. 17. But it seems to be impossible to determine the aerosol scattering phase function in this way, because not enough accurate initial data are used. In particular, insufficiently accurate data are used on the albedo of the underlying surface and the aerosol optical thickness in measurements of the sky brightness. Then, it is supposed that the reflection of radiation from the underlying surface follows the Lambert law, but, as a rule, it is far from reality. Besides, one should extrapolate the measurement data on the sky brightness phase functions to the scattering angles $\theta < 2°$ and $\theta > 160°$. The errors in reconstructing the aerosol scattering phase function by inverting the transfer equation that appear due to insufficiently accurate data were not estimated in Ref. 17. One of the aerosol scattering phase functions at $\lambda = 0.547$ μm obtained in such a way using data from Ref. 17 is presented in Table 2.

A great number of models of the aerosols scattering phase functions calculated by different authors for the aerosol of different microstructure can be found in the literature. The most known are the Tables of the Sky Brightness[18] which include the models of the aerosol scattering phase functions. One of the scattering phase functions[18] calculated using the values $\nu^* = 3.0$; $r_{min} = 0.04$ μm; $r_{max} = 10$ μm; $n = 1.5$; $\iota = 0$; and $\lambda = 0.55$ μm is presented in Table 2.

A number of the aerosols scattering phase functions calculated by the Mie theory for polymodal lognormal particle-size distribution at different heights up to 90 km and wavelength from 0.248 to 13.6 μm were published in 1982–1990 by G.M. Krekov et al. The scattering phase function we calculated by integration over height of the local scattering phase functions presented in Ref. 19 for the mean-cyclic conditions at $\lambda = 0.53$ μm is presented in Table 2 for a comparison.

All the scattering phase functions presented in Tables 1 and 2 are normalized according to the condition

$$\int_0^\pi \gamma_a(\theta, \lambda) \sin\theta \, d\theta = 2.$$

The scattering phase functions $\gamma_a(\theta)$ at the scattering angles $\theta = 1–10°$ and $90–180°$ are shown in Fig. 2 for a comparison.

The very important characteristic of the shape (asymmetry) of the scattering phase function is the asymmetry coefficient Γ_a:

$$\Gamma_a = \int_0^{\pi/2} \gamma_a(\theta, \lambda) \sin\theta \, d\theta \Big/ \int_{\pi/2}^{\pi} \gamma_a(\theta, \lambda) \sin\theta \, d\theta.$$

The values of the coefficient Γ_a for each scattering phase function are presented in the last rows of Tables 1 and 2.

One should pay special attention to a wide scatter of the values of the aerosol scattering phase function $\gamma_a(\theta, \lambda)$ and the asymmetry coefficients Γ_a presented in Table 2 and Fig. 2.

This scatter causes the urgency of constructing weighted mean aerosol scattering phase functions related to certain latitude and landscape–climatic conditions.

Based on the fact that the used bulk of data on the size distribution histograms and the refractive index of particles is related to mainly continental mid-latitude conditions and warm season, the weighted mean aerosol scattering phase functions presented in Table 1 can be recommended for use as the handbook models as being relevant under these conditions.

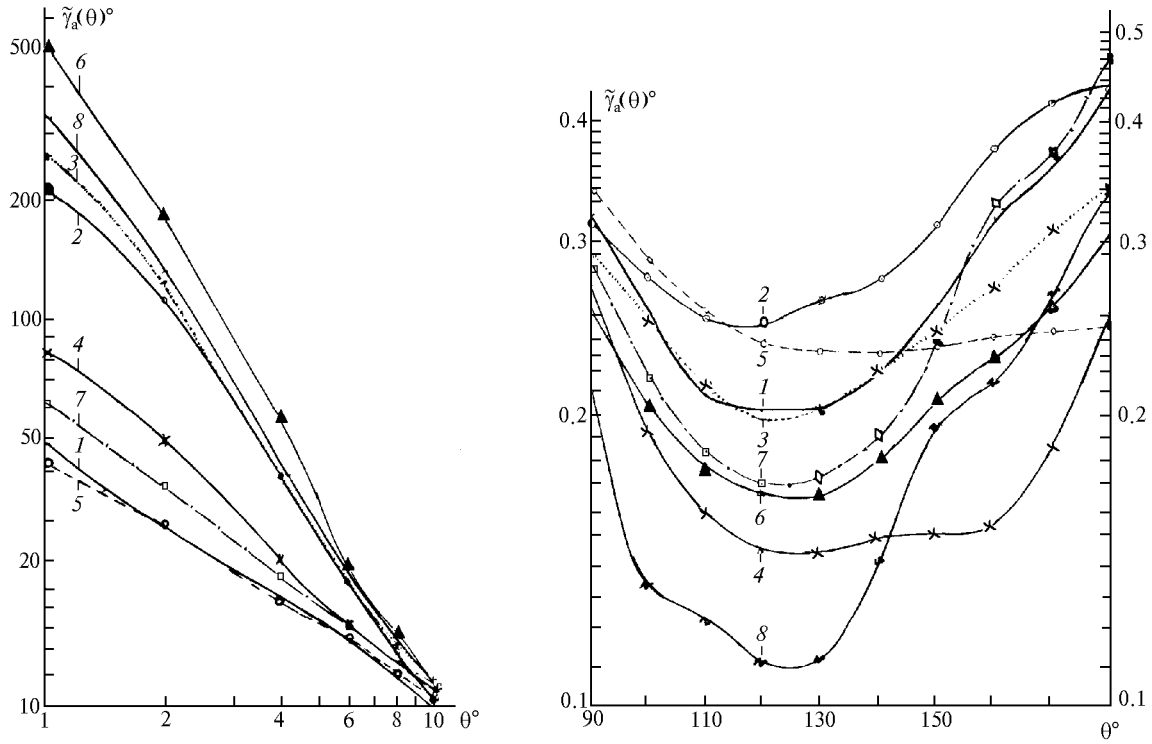

Fig. 2. Aerosol scattering phase functions $\gamma_a(\theta)$ at the scattering angles $\theta = 1–10$ and 90–180°: weighted mean scattering phase function ($\lambda = 0.55$ μm) (*1*), data from Ref. 10 (*2*); Ref. 11 (*3*); Refs. 14 and 15 (*4*); Refs. 14 and 16 (*5*); Ref. 17 (*6*); Ref. 18 (*7*) Ref. 19 (*8*).

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