SPATIAL FILTRATION OF HOLOGRAPHIC SHEAR INTERFEROGRAMS BASED ON THE USE OF A DIFFUSELY SCATTERED LIGHT FIELD

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A lateral shear interferometer based on a double–exposure recording of a hologram of the image produced by a light diffusely scattered by a mat screen and passed through a controllable convex lens is analyzed. It is shown theoretically and experimentally that spatial filtration in the hologram field enables one to control a lens over the field. The spatial filtering in the plane of the image of the mat screen makes it possible to record the interference pattern characterizing the phase distortions introduced in the reference wave by the aberrations in the optical system forming it.

The technique for producing the lateral shear interferograms in the bands of infinite width using a diffusely scattered radiation with spatial filtration in Refs. 1, 2, and 3 is described as an example of producing a doubly exposed hologram of a focused image of a mat screen. However, an increase of the focal length of the lens used for constructing the real image of the mat screen in the plane of a medium in which the hologram is recorded results in the increase of the dimensions of the holographic system. That, in turn, deteriorates its noiseproof. Partial reduction of the required dimensions can be achieved by recording a Fourier hologram of the mat screen.[4,5]

This paper considers a technique of the double–exposure recording of the holograms for producing by a diffusely scattered light field, of the lateral shear interferograms in the bands of infinite width for the quality control of the collecting lenses and objectives in a wide range of their focal lengths at a relatively small size of the holographic system.

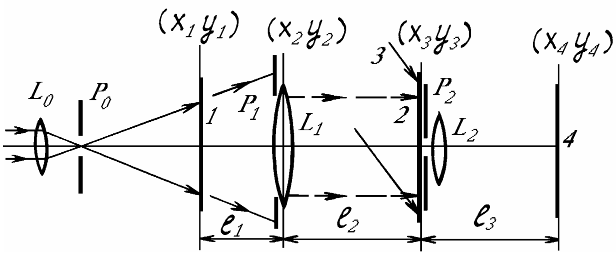

FIG. 1. The optical scheme used for recording and reconstructing a doubly exposed hologram: 1) mat screen; 2) photographic plate–hologram; 3) reference beam; 4) recording plane of the interferogram; L_0, L_1, and L_2 are lenses; p_1 and p_2 sre aperture diaphragms; and, p_0 is a spatial filter.

As shown in Fig. 1, the mat screen *1* is illuminated by an aberrationless diverging spherical wave with radius of curvature R formed by the lens L_0 and a point–size opening in the opaque screen P_0 placed at the focus of the lens. Then the diffusely scattered radiation passes through the controllable lens L_1 and a recording takes place during the first exposure of the photographic plate *2* with the use of the quasiplanar reference wave *3*. Prior to the second exposure, the mat screen is displaced in its plane by an amount a along the x axis while the photographic plate is displaced in the same direction by an amount b.

In the Fresnel approximation, neglecting constant factors, the complex amplitudes of the objective fields of the two exposures in the (x_3, y_3) plane of the photographic plate can be written in the form

$$u_1(x_3, y_3) \sim \int\int\int\int_{-\infty}^{\infty} t(x_1, y_1)\exp[ik(x_1^2 + y_1^2)/2R] \times$$
$$\times \exp\{ik[(x_1 - x_2)^2 + (y_1 - y_2)^2]/2l_1\}\, p_1(x_2, y_2) \times$$
$$\times \exp\{-i[k(x_2^2 + y_2^2)/2f_1] - \varphi_1(x_2, y_2)\} \times$$
$$\times \exp\{ik[(x_2 - x_3)^2 + (y_2 - y_3)^2]/2l_2\}\, dx_1dy_1dx_2dy_2 , \quad (1)$$

$$u_2(x_3, y_3) \sim \int\int\int\int_{-\infty}^{\infty} t(x_1 + a, y_1)\exp[ik(x_1^2 + y_1^2)/2R] \times$$
$$\times \exp\{ik[(x_1 - x_2)^2 + (y_1 - y_2)^2]/2l_1\}\, p_1(x_2, y_2) \times$$
$$\times \exp\{-i[k(x_2^2 + y_2^2)/2f_1] - \varphi_1(x_2, y_2)\} \times$$
$$\times \exp\{ik[(x_2 - x_3 + b)^2 + (y_2 - y_3)^2]/2l_2\}\, dx_1dy_1dx_2dy_2 , \quad (2)$$

where k is the wave number; $t(x_1, y_1)$ is the complex transmission amplitude of the mat screen, and a random function of the coordinates; $p_1(x_2, y_2)\exp i\varphi_1(x_2, y_2)$ is the generalized pupil function[6] of the controllable lens L_1 with focal length f_1, which accounts for its axial wave aberrations; l_1 and l_2 are the distances between the principal plane (x_2, y_2) of the lens L_1 and the planes of the mat screen and of the photographic plate, respectively.

If the condition $f_1 > l_1l_2/(l_1 + l_2)$ is satisfied, i.e., the lens L_1 forms a virtual image of the mat screen (see Fig. 1) and, in addition, $R = f_1 - l_1$, while the shifts of the mat screen and of the photographic plate satisfy the condition $b/a = f_1/(f_1 - l_1)$, then expressions (1) and (2) can be reduced to the form

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$$u_1(x_3, y_3) \sim \exp[ik(x_3^2 + y_3^2)(l_2 - L)/2l_2^2] \times$$
$$\times \{F[kx_3L/l_1l_2, ky_3L/l_1l_2] \otimes \exp[-ik(x_3^2 + y_3^2)\times$$
$$\times (f_1 - l_1)L/2f_1l_1l_2] \otimes P_1(x_3, y_3)\} ; \quad (3)$$

$$u_2(x_3, y_3) \sim \exp[ik(x_3^2 + y_3^2)(l_2 - L)/2l_2^2] \times$$
$$\times \exp(-ikaLx_3/l_1l_2)\{\exp(ikaLx_3/l_1l_2) \times$$
$$\times \{F[kx_3L/l_1l_2, ky_3L/l_1l_2] \otimes \exp[-ik(x_3^2 + y_3^2) \times$$
$$\times (f_1 - l_1)L/2f_1l_1l_2]\} \otimes P_1(x_3, y_3)\} ; \quad (4)$$

where $\otimes$ denotes the operation of convolution, $1/L = 1/l_1 - 1/f_1 + 1/l_2$, and

$$F[kx_3L/l_1l_2, ky_3L/l_1l_2] = \int\int_{-\infty}^{\infty} t(x_1, y_1) \times$$
$$\times \exp[-ik(x_1x_3 + y_1y_3)L/l_1l_2]\mathrm{d}x_1\mathrm{d}y_1$$

and

$$P_1(x_3, y_3) = \int\int_{-\infty}^{\infty} p_1(x_2, y_2) \exp i\varphi_1(x_2, y_2) \times$$
$$\times \exp[-ik(x_2x_3 + y_2y_3)/l_2]\mathrm{d}x_2\mathrm{d}y_2$$

are the Fourier transforms of the corresponding functions.

Let such a doubly exposed hologram be reconstructed by a copy of the reference wave corresponding to the first exposure. Then the diffraction field in its plane can be written as

$$u(x_3, y_3) \sim u_1(x_3, y_3) + u_2(x_3, y_3) \times$$
$$\times \exp i[\varphi_2(x_3, y_3) - \varphi_2(x_3 + b, y_3)] , \quad (5)$$

where $\varphi_2(x_3, y_3)$ is the deterministic phase function which characterizes the phase distortions introduced in the wave front of the reference wave by the aberrations of the optical system forming it.

Upon substituting expressions (3) and (4) into expression (5), assuming that the equality[7]

$$\exp(-ikaLx_3/l_1l_2)\{\exp(ikaLx_3/l_1l_2)\{F[kx_3L/l_1l_2, ky_3L/l_1l_2]\otimes$$
$$\otimes \exp[-ik(x_3^2 + y_3^2)(f_1 - l_1)L/2f_1l_1l_2]\} \otimes P_1(x_3, y_3)\} =$$
$$= F[kx_3L/l_1l_2, ky_3L/l_1l_2] \otimes \exp[-ik(x_3^2 + y_3^2) \times$$
$$\times (f_1 - l_1)L/2f_1l_1l_2]\} \otimes \exp(-ikax_3L/l_1l_2) P_1(x_3, y_3)$$

is valid, we obtain

$$u(x_3, y_3) \sim \exp[ik(x_3^2 + y_3^2)(l_2 - L)/2l_2^2] \times$$
$$\times \{F[kx_3L/l_1l_2, ky_3L/l_1l_2] \otimes \exp[-ik(x_3^2 + y_3^2) \times$$
$$\times (f_1 - l_1)L/2f_1l_1l_2] \otimes P_1(x_3, y_3) + \exp i[\varphi_2(x_3, y_3) -$$
$$-\varphi_2(x_3 + b, y_3)]\{F[kx_3L/l_1l_2, ky_3L/l_1l_2] \otimes \exp[-ik(x_3^2 + y_3^2)\times$$
$$\times (f_1 - l_1)L/2f_1l_1l_2] \otimes \exp(-ikax_3L/l_1l_2) P_1(x_3, y_3)\}\} . \quad (6)$$

As follows from expression (6), the identical subjective speckles of both exposures with their relative tilt angle $\alpha = aL/l_1l_2$ coincide in the hologram plane, therefore the low frequency interference pattern produced due to the phase distortions of the reference wave is localized in this plane.[2] If an opaque screen with a circular opening (the aperture diaphragm p_2 of the lens L_2 in Fig. 1) centered on the optical axis is placed in the hologram plane and the condition $\varphi_2(x_3, y_3) - \varphi_2(x_3 + b, y_3) \le \pi$ is satisfied within the diameter of the opening, i.e., the diameter of the filtering opening does not exceed the width of the interference band for the interference pattern localized in the hologram plane, then the diffraction field can be represented by the expression

$$u(x_3, y_3) \sim p_2(x_3, y_3) \exp[ik(x_3^2 + y_3^2)(l_2 - L)/2l_2^2] \times$$
$$\times\{F[kx_3L/l_1l_2, ky_3L/l_1l_2] \otimes \exp[-ik(x_3^2 + y_3^2)(f_1 - l_1)L/2f_1l_1l_2]\}\otimes$$
$$\otimes[1 + \exp(-ikax_3L/l_1l_2)] P_1(x_3, y_3)\} , \quad (7)$$

where $p_2(x_3, y_3)$ is the transmission function of the screen with circular opening.[8]

The complex amplitude of the field at the distance l_3 from the hologram plane can be written, in the approach being used as follows:

$$u(x_4, y_4) \sim \int\int_{-\infty}^{\infty} u(x_3, y_3) \exp[-ik(x_3^2 + y_3^2)/2f_2] \times$$
$$\times \exp\{ik[(x_3 - x_4)^2 + (y_3 - y_4)^2]/2l_3\}\mathrm{d}x_3\mathrm{d}y_3 , \quad (8)$$

where f_2 is the focal length of the lens L_2 (see Fig. 1).

Upon substituting expression (7) into expression (8), assuming that the condition $1/f_2 = (l_2 - L)/l_2^2 + 1/l_3$ is satisfied, we obtain

$$u(x_4, y_4) \sim \exp[ik(x_4^2 + y_4^2)/2l_3] \{t(-\mu_1x_4, -\mu_1y_4) \times$$
$$\times \exp[ik(x_4^2 + y_4^2)f_1l_1l_2/2(f_1 - l_1)Ll_3^2] \times$$
$$\times p_1(-\mu_2x_4, -\mu_2y_4) \exp i\varphi_1(-\mu_2x_4, -\mu_2y_4) \otimes P_2(x_4, y_4) +$$
$$+ t(-\mu_1x_4 - \mu_1y_4) \exp[ik(x_4^2 + y_4^2)f_1l_1l_2/2(f_1 - l_1)Ll_3^2] \times$$
$$\times p_1(-\mu_2x_4 - aL/l_1, -\mu_2y_4) \times$$
$$\times \exp i\varphi_1(-\mu_2x_4 - aL/l_1, -\mu_2y_4) \otimes P_2(x_4, y_4)\} , \quad (9)$$

where $\mu_1 = l_1l_2/Ll_3$ and $\mu_2 = l_2/l_3$ are the scale factors of image transformation and

$$P_2(x_4, y_4) = \int\int_{-\infty}^{\infty} p_2(x_3, y_3) \exp[-ik(x_3x_4 + y_3y_4)/l_3]\mathrm{d}x_3\mathrm{d}y_3$$

is the Fourier transform of the transmission function of the filtering opening.

From expression (9) it follows that the image of the mat screen limited by the aperture size of the lens L_1 is formed in the (x_4, y_4) plane if the diameter of the illuminated zone exceeds $d_1 l_1/L$, where d_1 is the diameter of the pupil of the lens L_1. Each point of the image spreads in this case to the dimension of the Fourier transform of the transmission function of the lens L_2, which determines the characteristic size of the subjective speckle. In addition, within the region of overlap of the images of the pupil of the lens L_1, the identical speckle fields of the two exposures are superimposed on one another which results in the localization of the low frequency interference pattern in the recording plane *4* (see Fig. 1). Actually, if the period of the function $\exp i\varphi_1(-\mu_2 x_4, -\mu_2 y_4) + \exp i\varphi_1(-\mu_2 x_4 - aL/l_1, -\mu_2 y_4)$ is at least an order of magnitude greater than the size of a speckle[9] then in expression (13) that function can be removed from the integrand of the convolution integral. Then the superposition of the correlated speckle fields of the two exposures yields the distribution of the irradiance

$$I(x_4, y_4) \sim \Big\{1 + \cos\big[(\varphi_1(-\mu_2 x_4, -\mu_2 y_4) -$$
$$- \varphi_2(-\mu_2 x_4 - aL/l_1, -\mu_2 y_4)\big]\Big\} \Big| t(-\mu_1 x_4, -\mu_1 y_4) \times$$
$$\times \exp\big[ik(x_4^2 + y_4^2) f_1 l_1 l_2 / 2(f_1 - l_1) L l_3^2\big] \otimes P_2(x_4, y_4)\Big|^2. \quad (10)$$

Thus, one can see from expression (10) that the image of the mat screen is modulated not only by a random speckle structure but also by a low frequency regular intereference pattern, which has the form of a lateral shear interferogram in the bands of infinite width characterizing the axial wave aberrations of the lens L_1. This can be explained by the fact that the information on the phase distortions introduced in the light wave by the controllable lens, is contained within an individual speckle in the hologram plane. Hence it follows that by performing the spatial filteration on the optical axis in the plane of the hologram, a narrow spatial frequency range can be selected from the spatial spectrum of waves scattered by the mat screen around the direction of the optical axis. At the same time, a shift of the filtering screen along the x axis in the hologram plane results in the formation of the lateral shear interferogram in the bands of infinite width which in a combined way characterises the on–axis and off–axis wave aberrations due to the controllable lens L_1 because in this case the filtering opening selects a narrow spatial frequency range around the direction corresponding to the spatial frequency $x_{30}(l_2 - L)/\lambda l_2^2$, where λ is the wavelength of light used for recording and reproducing the hologram, x_{30} is the coordinate of the center of the filtering opening in the opaque screen p_2 (see Fig. 1).

To record the interference pattern localized in the hologram plane let us perform the spatial filtering of a diffusely scattered field in the image plane (x_4, y_4) of the mat screen (see Fig. 2). By substituting expression (6) into expression (8), neglecting the diffraction limitations due to the finite dimensions of the hologram and the lens L_2, we obtain

$$u(x_4, y_4) \sim \exp\big[ik(x_4^2 + y_4^2)/2l_3\big] \big\{t(-\mu_1 x_4, -\mu_1 y_4) \times$$
$$\times \exp\big[ik(x_4^2 + y_4^2) f_1 l_1 l_2 / 2(f_1 - l_1) L l_3^2 \times$$
$$\times p_1(-\mu_2 x_4, -\mu_2 y_4) \exp i\varphi_1(-\mu_2 x_4, -\mu_2 y_4) +$$
$$+ \Phi(x_4, y_4) \otimes t(-\mu_1 x_4, -\mu_1 y_4) \times$$
$$\times \exp\big[ik(x_4^2 + y_4^2) f_1 l_1 l_2 / 2(f_1 - l_1) L l_3^2\big] \times$$
$$\times p_1(-\mu_2 x_4 - aL/l_1, -\mu_2 y_4) \exp i\varphi_1(-\mu_2 x_4, -\mu_2 y_4)\big\}, \quad (11)$$

where

$$\Phi(x_4, y_4) = \int\int_{-\infty}^{\infty} \exp i\big[\varphi_2(x_3, y_3) - \varphi_2(x_3 + b, y_3)\big] \times$$
$$\times \exp[-ik(x_3 x_4 + y_3 y_4)/l_3] \mathrm{d}x_3 \mathrm{d}y_3$$

is the Fourier transform of the corresponding function.

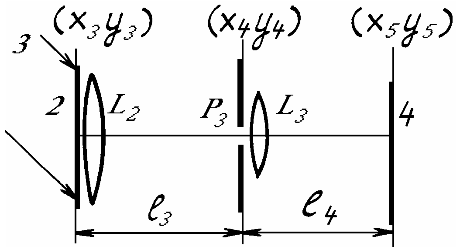

FIG. 2. The optical scheme used for recording of the interference pattern localized in the hologram plane when spatial filtering is performed in the image plane of the mat screen.

If the opaque screen with a circular opening (the aperture diaphragm p_3 of the lens L_3 in Fig. 2) centered on the optical axis is placed in the (x_4, y_4) plane and the width of an intereference band of the interference pattern localized in the image plane of the mat screen does not exceed the diameter of the filtering opening, i.e., $\varphi_1(-\mu_2 x_4 - aL/l_1, -\mu_2 y_4) - \varphi_1(-\mu_2 x_4, -\mu_2 y_4) \le \pi$, then the diffraction field at the exit from the filtering opening is

$$u(x_4, y_4) \sim p_3(x_4, y_4) \exp\big[ik(x_4^2 + y_4^2)/2l_3\big] \big\{t(-\mu_1 x_4, -\mu_1 y_4) \times$$
$$\times \exp\big[ik(x_4^2 + y_4^2) f_1 l_1 l_2 / 2(f_1 - l_1) L l_3^2\big] \otimes [1 + \Phi(x_4, y_4]\big\}, \quad (12)$$

where $p_3(x_4, y_4)$ is the transmission function of the screen p_3 (see Fig. 2) with a circular opening.

Let the light field in the (x_5, y_5) plane located at the distance l_4 from the plane of filtration be

$$u(x_5, y_5) \sim \int\int_{-\infty}^{\infty} u(x_4, y_4) \exp\big[-ik(x_4^2 + y_4^2)/2f_3\big] \times$$
$$\times \exp\{ik[(x_4 - x_5)^2 + (y_4 - y_5)^2]/2l_4\} \mathrm{d}x_4 \mathrm{d}y_4, \quad (13)$$

where f_3 is the focal length of the lens L_3.

Assuming that the condition $1/l_3 + 1/l_4 = 1/f_3$ is satisfied, we obtain upon substituting expression (12) into expression (13)

$$u(x_5, y_5) \sim \exp\big[-ik(x_5^2 + y_5^2)/2l_4\big] \{\{1 + \exp i\big[\varphi_2(-\mu_3 x_5, -\mu_3 y_5) -$$
$$- \varphi_2(-\mu_3 x_5 + b, -\mu_3 y_5)\big]\} F[kx_5/l_4, ky_5/l_4] \otimes P_3(x_5, y_5)\}, \quad (14)$$

where $\mu_3 = l_3/l_4$ is the scale factor of the transformation and

$$F[kx_5/l_4,\ ky_5/l_4] = \int\int_{-\infty}^{\infty} \{t(-\mu_1 x_4,\ -\mu_1 y_4) \times$$

$$\times \exp[ik(x_4^2 + y_4^2)f_1 l_1 l_2/2(f_1 - l_1)Ll_3^2] \times$$

$$\times \exp[-ik(x_4 x_5 + y_4 y_5)/l_4]\, dx_4 dy_4\ ;$$

and

$$P_3(x_5,\ y_5) = \int\int_{-\infty}^{\infty} p_3(x_4,\ y_4) \exp[-ik(x_4 x_5 + y_4 y_5)/l_4]\, dx_4 dy_4$$

are the Fourier transforms of the corresponding functions.

If the period of the function $1 + \exp i[\varphi_2(-\mu_3 x_5,\ -\mu_3 y_5) - \varphi_2(-\mu_3 x_5 + b,\ -\mu_3 y_5)]$ exceeds the characteristic size of a speckle determined by the width of the function $P_3(x_5,\ y_5)$, then this function can be removed from the integrand of the integral of concolution in expression (14). Then the superposition of the correlated speckle fields of the two exposures yields the distribution of the irradiance

$$I(x_5,\ y_5) \sim \{1 + \cos[\varphi_2(-\mu_3 x_5,\ -\mu_3 y_5) -$$

$$- \varphi_2(-\mu_3 x_5 + b, -\mu_3 y_5)]\} \,|\, F[kx_5/l_4,\ ky_5/l_4] \otimes P_3(x_5,\ y_5)\} \,|^2, \quad (15)$$

which describes the speckle structure modulated by the interference bands. The interference pattern has the form of a lateral shear interferogram in the bands of infinite width. It characterizes the wave aberrations of the optical system forming the wave front of the reference wave. A shift of the filtering opening from the optical axis results in a lower contrast of the interference pattern as well as in the change of its form due to the off–axis wave aberrations produced by the controllable lens.

In accordance with expression (6), there is a common quadratic factor in the distributions of the complex amplitude of the fields of the two exposures in the hologram plane in the minus–first diffraction order. This factor characterizes the distribution of the phase of a diverging spherical wave with radius of curvature $l_2^2/(l_2 - L)$. Therefore, the correcting lens L_2 (see Fig. 2) must be placed in the hologram plane for recording the interference pattern localized in it. In the case of reconstruction of the hologram in the plus–first diffraction order, the real image of the mat screen is formed at the distance $l_2^2/(l_2 - L)$ from the hologram plane, and the spatial fltration on the optical axis in that plane with the help of the aperture diaphragm of the collecting lens or objective is needed to record the interference pattern localized in the hologram plane.

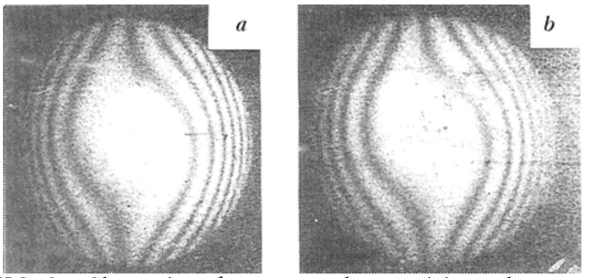

FIG. 3. Shear interferograms characterizing the wave aberrations of a controllable object with the spatial filtration in the hologram plane: a) on the optical axis and b) off the optical axis.

In experiments, the double–exposure holograms were recorded on the photographic plates of the type Mikrat–VRL using a He–Ne laser at the wavelength 0.63 μm. As an example Fig. 3*a* shows a lateral shear interferogram characterizing the spherical aberration in the paraxial focus of the objective 300 mm in focal length and the pupil 50 mm in diameter. Spatial filtration was performed on the optical axis by reconstructing the hologram with a narrow (3 mm in diameter) laser beam. The hologram was recorded at $l_1 = 85$ mm and $l_2 = 200$ mm. Diameter of the illuminated spot on the mat screen was 60 mm, while the reference beam diameter was 50 mm. Prior to the second exposure the mat screen was shifted by 1.3 ± 0.002 mm while the photographic plate – by 1.349 ± 0.002 mm. The 15 mm shift of the hologram with respect to the laser beam reconstructing it in the direction of the hologram shift prior to the second exposure resulted in the formation of the interference pattern in the image plane of the mat screen which is shown in Fig. 3*b* and characterizes in a combinated way the on–axis and off–axis wave aberrations due to the controllable objective.

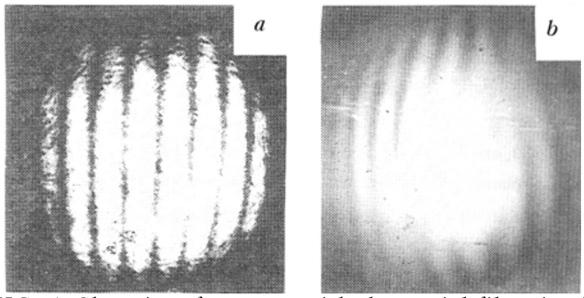

FIG. 4. Shear interferograms with the spatial filtration in the image plane of the mat screen: a) on the optical axis and b) off the optical axis.

The lateral shear interferogram in the bands of infinite width (see Fig. 4*a*) characterizes the spherical aberration with the prefocal defocusing of the reference wave. This interferogram was recorded by reconstructing the hologram in the plus–first diffraction order and performing the spatial filtration on the optical axis in the plane of the real image of the mat screen with the help of the aperture diaphragm (2 mm in diameter) of the objective. The 5 mm shift of the aperture diaphragm from the optical axis yields the interference pattern shown in Fig. 4*b*. By comparing the interference patterns in Fig. 4*a* and Fig. 4*b*, one can see that the off–axis spatial filtration results in the lower contrast of the interference pattern and in the change of its form. This fact can be explained by the effect of the off–axis wave aberrations of the controllable objective, as has been mentioned above, and must be analyzed in ample detail. To this end, in order to simplify formulas, let us assume that the controllable lens or objective is placed in the plane of the mat screen (i.e., $l_1 = 0$), and the optical system (see Fig. 2) has a unit magnification. Under these assumptions the complex amplitude of the field in the recording plane $(x_5,\ y_5)$ takes the form

$$u(x_5,\ y_5) \sim \exp[ik(x_5^2 + y_5^2)/2l_2]\, \{F_1(x_5,\ y_5) \otimes P(x_5,\ y_5) +$$

$$+ \exp i[\varphi_2(-x_5,\ -y_5) - \varphi_2(-x_5 + a,\ -y_5)]\, F_2(x_5,\ y_5) \otimes P(x_5,\ y_5)\},$$

where

$$F_1(x_5,\ y_5) = \int\int_{-\infty}^{\infty} t(-x_4,\ -y_4) \exp[ik(x_4^2 + y_4^2)/2l_2] \times$$

$$\times \exp i\varphi_1(-x_4,\ -y_4) \exp[-ik(x_4 x_5 + y_4 y_5)/l_2]\, dx_4 dy_4$$

and

$$F_2(x_5, y_5) = \int_{-\infty}^{\infty}\int t(-x_4, -y_4)\exp[ik(x_4^2 + y_4^2)/2l_2] \times$$

$$\times \exp i[\varphi_1(-x_4 - a, -y_4)\exp[-ik(x_4x_5 + y_4y_5)/l_2]\,\mathrm{d}x_4\mathrm{d}y_4 ;$$

$$P(x_5, y_5) = \int_{-\infty}^{\infty}\int p_3(x_4, y_4)\exp[-ik(x_4x_5 + y_4y_5)/l_2]\,\mathrm{d}x_4\mathrm{d}y_4$$

are the Fourier transforms of the corresponding functions.

Let us now write down an expression for the distribution of light intensity in the recording plane. In order to eliminate the speckle effect, let us introduce averaging over the coordinates assuming the area of averaging to be much larger than the size of an individual speckle being at the same time small enough for the phase factor $\exp i[\varphi_2(-x_5, -y_5) - \varphi_2(-x_5 + a, -y_5)]$ to remain constant within it. Then for the average intensity we obtain

$$\langle I(x_5, y_5)\rangle = \langle|F_1(x_5, y_5)\otimes P(x_5, y_5)|^2\rangle +$$
$$+ \langle|F_2(x_5, y_5)\otimes P(x_5, y_5)|^2\rangle +$$
$$+ 2\{\mathrm{Re}\langle[F_1(x_5, y_5)\otimes P(x_5, y_5)]\,[F_2(x_5, y_5)\otimes P(x_5, y_5)]^*\rangle\} \times$$
$$\times \cos[\varphi_2(-x_5, -y_5) - \varphi_2(-x_5 + a, -y_5)] -$$
$$- 2\{\mathrm{Im}\langle[F_1(x_5, y_5)\otimes P(x_5, y_5)]\,[F_2(x_5, y_5)\otimes P(x_5, y_5)]^*\rangle\} \times$$
$$\times \sin[\varphi_2(-x_5, -y_5) - \varphi_2(-x_5 + a, -y_5)] ,$$

where Re and Im denote the real and imaginary parts of the quantity, and angular brackets denote averaging over the coordinates. Assuming average intensities of the reconstructed fields corresponding to the first and second exposures to be identical, we represent the function $\langle I(x_5, y_5)\rangle$ in the form

$$\langle I(x_5, y_5)\rangle = 2\langle|F_1(x_5, y_5)\otimes P(x_5, y_5)|^2\rangle \times$$
$$\times \{1 + |V|\cos[\varphi_2(-x_5, -y_5) - \varphi_2(-x_5 + a, -y_5) + \psi]\} ,$$

where

$$V = \frac{\langle[F_1(x_5, y_5)\otimes P(x_5, y_5)]\,[F_2(x_5, y_5)\otimes P(x_5, y_5)]^*\rangle}{\langle|F_1(x_5, y_5)\otimes P(x_5, y_5)|^2\rangle}$$

is the normalized correlation function and $\psi = \arg V$.

The function characterizing only the first order axial wave aberrations produced by the controllable lens or objective is given by

$$\varphi_1(x_4, y_4) = A(x_4^2 + y_4^2)^2 + E(x_4^2 + y_4^2),$$

where A and E are the coefficients characterizing the spherical aberration and defocusing, respectively. In addition, we will assume that the interference pattern localized in the image plane of the mat screen is produced only by defocusing, and the autocorrelation function

$$\langle t(-x_4, -y_4)\,t^*(-x_4', -y_4')\rangle = \delta(x_4 - x_4')\,\delta(y_4 - y_4') ,$$

where δ is the delta function. Then the formula for the correlation function takes the form

$$V = \frac{\int_{-\infty}^{\infty}\int |p_3(x_4, y_4)|^2\exp(-ikaEx_4)\mathrm{d}x_4\mathrm{d}y_4}{\int_{-\infty}^{\infty}\int |p_3(x_4, y_4)|^2\mathrm{d}x_4\mathrm{d}y_4} .$$

This formula is analogous to the well–known relation of holographic interferometry[10] using diffusely scattered fields, and determines the normalized autocorrelation function of speckle fields in the recording plane (x_5, y_5) of the interference pattern, i.e., it is a normalized complex coefficient of coherence.

Since the transmission function of the screen with circular opening equals unity within the opening and vanishes outside it, the quantity $|p_3(x_4, y_4)|^2 = p_3(x_4, y_4)$ and the normalized autocorrelation function takes the form

$$V = \frac{\int_{-\infty}^{\infty}\int p_3(x_4, y_4)\exp(-ikaEx_4)\mathrm{d}x_4\mathrm{d}y_4}{\int_{-\infty}^{\infty}\int p_3(x_4, y_4)\,\mathrm{d}x_4\mathrm{d}y_4} ,$$

i.e., the autocorrelation function is determined as the normalized Fourier transform of the transmission function of the screen with circular opening. Then the contrast of the interference pattern is $|V| = |(2J_1(kd_3aE/2)/kd_3aE/2)|$, where J_1 is the first order Bessel function of the first kind and d_3 is the diameter of the filtering opening in the plane (x_4, y_4) in Fig. 2. If the width $\Delta x_4 = \lambda/aE$ of an interference band of an interference pattern localized in the image plane of the mat screen in the form of equidistant bands exceeds, in this simplified case, the diameter of the filtering opening, then $|V| \simeq 1$ and $\psi = 0$ for the interference pattern being recorded in the plane (x_5, y_5). The same results will be obtained also in the case of spherical aberration produced by the controllable lens or objective if the width of an interference band in the (x_4, y_4) plane exceeds the diameter of the filtering opening.

In accordance with the recommendation given in Ref. 11, let us write down, neglecting the distortion which is not determined in the differential interferometry, the function characterizing the first order off–axis wave aberrations produced by the controllable lens or thes objective in the form

$$\varphi_1(x_4, y_4) = B(\xi x_4 + \eta y_4)(x_4^2 + y_4^2) + C(\xi^2 + \eta^4) \times$$
$$\times (x_4^2 + y_4^2) + D[(\xi^2 - \eta^4)(x_4^2 - y_4^2) + 4\xi\eta x_4y_4]$$

where B, C, and D are the coefficients characterizing the coma, curvature of the field, and asigmatism, respectively, where $\xi = x_3/\lambda l_2$ and $\eta = y_3/\lambda l_2$ are the spatial frequencies. Hence it follows that the contrast and the form of the interference pattern recorded in the (x_5, y_5) plane will be independent of the off–axis wave aberrations of the controllable lens or objective, when the spatial filtering is performed on the optical axis in the image plane of the mat screen provided the diameter of the filtering opening is sufficiently small $(x_4 = y_4 = 0)$. A shift of the filtering opening from the optical axis in the plane (x_4, y_4) yields the following form of the correlation function:

$$V = \frac{\int\limits_{-\infty}^{\infty}\int p_3(x_4 + a_0,\ y_4 + b_0)\exp\left[-i\,\frac{\partial\varphi_1(-x_4,\ -y_4)}{\partial x_4}\,a\right]\mathrm{d}x_4\mathrm{d}y_4}{\int\limits_{-\infty}^{\infty}\int p_3(x_4 + a_0,\ y_4 + b_0)\mathrm{d}x_4\mathrm{d}y_4},$$

where a_0 and b_0 are the coordinates of the center of the filtering opening. As a consequence, even at a sufficiently small diameter of the filtering opening providing the selection of the narrow spectral band centered at the spatial frequency ρ_0^2 is equal to $\xi_1^2 + \eta_1^2$, where $\xi_1 = a_0/\lambda l_2$ and $\eta_1 = b_0/\lambda l_2$, $|V| < 1$, and $\psi \neq 0$ for the interference pattern recorded in the (x_5, y_5) plane because of the square law dependence of the exponent in the expression for the correlation function in the case of off–axis coma aberration.

Thus, the above–given theoretical and experimental results showed that the proposed technique for a double–exposure recording of the hologram produced by a diffusely scattered light field allows one to form the lateral shear interferograms in the bands of infinite width. In this case the spatial filtering in the plane of the hologram is needed for recording of the interference pattern characterizing the wave aberrations of the controllable lens or the objective localized in the image plane of the mat screen. The interference pattern due to the phase distortions of the reference wave produced by the aberrations in the optical system forming its wave front is localized in the hologram plane, and spatial filtration on the optical axis in the image plane of the mat screen is needed to record it. It should be noted in addition that $l_1 = 0$ the technique eliminates increasing the frequency of the interference bands due to a finite accuracy of the shifts of the mat screen and the photographic plate prior to the second exposure using one and the same mechanism.

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