APPLICABILITY LIMITS OF THE TRANSFER EQUATION TO DESCRIPTION OF SPONTANEOUS RADIATION AMPLIFICATION IN AN INHOMOGENEOUS ACTIVE MEDIUM

V.V. Kolosov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
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This paper describes an analytical solution of the exact equation for coherence function for parabolic profiles of the refractive index and amplification coefficient distribution in an active medium cross-section. The applicability limits of the radiation transfer equation are analyzed by a direct comparison of analytical solutions of the equation for coherence function and the radiation transfer equation for a wide range of the active medium dimensionless parameters. The problem of the applicability limits is also discussed based on a comparative analysis of general solutions of the exact and approximate integral equations.

The investigation of spontaneous radiation amplification in active media with inhomogeneous distribution of inverse population is important for the creation of lasers without cavity, i.e., lasers without end reflectors (mirrors). There are several approaches to the theoretical investigation of output radiation of such lasers. The approaches are based on the paraxial approximation. As an original equation either a wave equation

$$2ik\frac{\partial E}{\partial z} + \nabla_{\perp}^2 E + k^2\,\Delta\varepsilon(z,\,\rho)\,E(z,\,\rho) = P_{\rm sp}(z,\,\rho)\,, \quad (1)$$

is used or identical equation for the second order coherence function ($\mathbf{R} = (\rho_1 + \rho_2)/2$, $\rho = \rho_1 - \rho_2$)

$$2ik\frac{\partial \Gamma_2}{\partial z} + 2\nabla_{\mathbf{R}}\nabla_{\rho}\Gamma_2 + k^2\,[\Delta\varepsilon(z,\,\mathbf{R}+\rho/2) -$$
$$-\,\Delta\varepsilon\,(z,\,\mathbf{R}-\rho/2)]\,\Gamma_2(z,\,\mathbf{R},\,\rho) = \frac{iW_{\rm ef}}{k}\,g(z,\,\mathbf{R})\,\delta(\rho). \quad (2)$$

As a rule, the authors (for example, Refs. 1 and 2) when solving Eq. (2) use the Taylor approximation

$$\Delta\varepsilon(z,\,\mathbf{R}+\rho/2) - \Delta\varepsilon^*\,(z,\,\mathbf{R}-\rho/2) =$$
$$= \varepsilon\,(z,\,\mathbf{R}+\rho/2) - \varepsilon\,(z,\,\mathbf{R}-\rho/2) +$$
$$+\, i\,[\sigma\,(z,\,\mathbf{R}+\rho/2) + \sigma\,(z,\,\mathbf{R}-\rho/2)] \cong$$
$$\cong \rho\nabla_{\mathbf{R}}\,\varepsilon\,(z,\,\mathbf{R}) + 2i\,\sigma\,(z,\,\mathbf{R}) + i\left(\frac{\mathbf{r}}{2}\nabla_{\mathbf{R}}\right)^2\sigma\,(z,\,\mathbf{R})\,. \quad (3)$$

It should be noted that the expansion (3) is exact for a parabolic profile of complex dielectric constant. Then, omitting the last term of Eq. (3) and substituting the result in Eq.(2) we derive the following equation

$$\frac{\partial\Gamma_2}{\partial z} + \left[\frac{1}{ik}\nabla_{\rho}\nabla_{\mathbf{R}} + \frac{k}{2i}\rho\nabla_{\mathbf{R}}\,\varepsilon(z,\,\mathbf{R}) + k\,\sigma(z,\,\mathbf{R})\right]\Gamma_2(z,\,\mathbf{R},\,\rho) =$$
$$= \frac{W_{\rm ef}}{2k^2}\,g(z,\,\mathbf{R})\,\delta(\rho). \quad (4)$$

This equation is already an approximate consequence of the equation for the transverse coherence function (2). The above approximations are justified from the point of view that the obtained equation makes it possible both the creation of effective numerical algorithms for this problem solution and the derivation of analytical solution for inhomogeneous distribution of dielectric constant and amplification factor in active media.[1–4]

However, as mentioned above,[3] for media with transverse inhomogeneity of the amplification (absorption) factor Eq. (4) well describes the power radiation characteristics and introduce an error in the estimate of its coherent properties. In particular, in this case we would have to use the following definition of a module of the coherence degree:

$$\mu(\rho) = \frac{|\Gamma_2(z,\,\mathbf{R}=0,\,\rho)|}{W(z,\,\mathbf{R}=0)} = \frac{|\Gamma_2(z,\,\mathbf{R}=0,\,\rho)|}{\Gamma_2(z,\,\mathbf{R}=0,\,\rho=0)} \quad (5)$$

instead of the exact definition

$$\mu(\rho) = \frac{|\Gamma_2(z,\,\mathbf{R}=0,\,\rho)|}{\sqrt{W(z,\,\mathbf{R}=\rho/2)\,W(z,\,\mathbf{R}=-\,\rho/2)}}\,, \quad (6)$$

since the latter gives rise to incorrect results. This fact calls for further theoretical analysis of the applicability limits of this equation to the problems of such a kind.

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Analysis of the applicability limits is performed by direct comparison of analytical solutions of the exact equation for the coherence function (2) and the approximate equation (4). Solutions of these equations are obtained for the parabolic profile of distribution of the refractive index and the amplification factor in the active medium cross-section without considering the effect of dielectric constant fluctuations, i.e., it is believed that

$$\varepsilon(\mathbf{R}) = 1 + (\mathbf{R}^2 - a^2)/L_\mathrm{R}^2,\ |\mathbf{R}| < a,\ \ \varepsilon(\mathbf{R}) = 1,\ |\mathbf{R}| > a, \quad (7)$$

$$g(\mathbf{R}) = g_0(1 - \mathbf{R}^2/a^2),\ |\mathbf{R}| < a,\ \ g(\mathbf{R}) = 0,\ |\mathbf{R}| > a,$$

where $L_\mathrm{R}^2 = a^2/\varepsilon_0$ is the refraction length, ε_0 and g_0 are the perturbations of the dielectric constant and the amplification factor on the beam axis, an imaginary part of the dielectric constant σ is connected with the amplification factor of a medium g by the following ratio:

$$\sigma(z, \mathbf{R}) = -k^{-1} g(z, \mathbf{R}).$$

Spontaneous radiation depends upon the presence of random polarization in a medium, which is considered to be of Gaussian Statistics and satisfies the condition

$$\langle P_\mathrm{sp}(\mathbf{r})\, P^*_\mathrm{sp}(\mathbf{r}')\rangle = W_\mathrm{ef}(\mathbf{r})\, g_0\, \delta(\mathbf{r} - \mathbf{r}'), \quad \mathbf{r} = \{z, \mathbf{R}\}, \quad (8)$$

where W_ef is the effective intensity of spontaneous emission. The condition (8) was used when deriving Eqs. (2) and (4), and it will be needed in what follows. Besides, the following approximations

$$W_\mathrm{ef}(z, \mathbf{R}) = W_{\delta\,\mathrm{ef}}(\mathbf{R})\, \delta(z), \quad (9)$$

$$W_{\delta\,\mathrm{ef}}(\mathbf{R}_0) = W_{\delta 0} \exp(-R_0^2/a^2) \quad (10)$$

were used.

The condition (9) denotes that we consider the contribution to the output radiation only from an infinitely thin layer of emitters located in the end area of the active medium (approximation of incoherent disc). The condition (10) can also be considered as an approximation, since in a rigorous statement of the problem the source intensity distribution must repeat the amplification factor distribution, i.e., it must have the parabolic profile.

Based on these assumptions, in Ref. 3, we obtain the solution of the approximate equation (4), from which it follows that the intensity distribution and the radiation coherence function in the output plane are of the form:

$$W(z, \mathbf{R}) = \Gamma_2(z, \mathbf{R}, \rho = 0) = \frac{W_{\delta\,\mathrm{ef}}}{8\pi} \frac{a^2 g_0 z_\mathrm{ef} \exp(g_0 z)}{L_\mathrm{R}^2 \mathrm{sh}^2(\bar z)(\bar a_0^{-2} + A)} \exp\left(-\frac{\mathbf{R}^2}{a_\mathrm{w}^2}\right),$$

$$\Gamma_2(z, \mathbf{R} = 0, \rho) = \frac{W_{\delta\,\mathrm{ef}}}{8\pi} \frac{g_0 z_\mathrm{ef} \exp(g_0 z)\, a^2}{L_\mathrm{R}^2 \mathrm{sh}^2(\bar z)(\bar a_0^{-2} + A)} \exp\left(-\frac{\rho^2}{4 a_\rho^2}\right), \quad (11)$$

$$a_\mathrm{w}^2 = a^2(\bar a_0^{-2} + A)/(\bar a_0^{-2} A + A^2 - B^2),$$

$$a_\rho^2 = a^2(\bar a_0^{-2} + A)/C^2, \quad \bar z = z/L_R,$$

$$\bar a_0 = a_0/a; \quad z_\mathrm{ef} = 1/g_0,$$

$$A = \frac{g_0 L_\mathrm{R}}{2} \frac{\mathrm{sh}(\bar z)\,\mathrm{ch}(\bar z) - \bar z}{\mathrm{sh}^2(\bar z)},$$

$$B = \frac{g_0 L_\mathrm{R}}{2} \frac{\mathrm{sh}(\bar z) - \bar z\,\mathrm{ch}(\bar z)}{\mathrm{sh}^2(\bar z)}, \quad C = \frac{L_\mathrm{D}}{L_\mathrm{R}\,\mathrm{sh}(\bar z)},$$

$$\mu(\rho) = |\Gamma_2(z, \mathbf{R} = 0, \rho)|/W(z, \mathbf{R} = 0) = \exp\left(-\frac{\rho^2}{4a_\rho^2}\right).$$

In the framework of the above assumptions (7)–(10) it is possible to obtain the solution of the exact equation (2). For this purpose we use the so-called method of complex ABCD radiation matrix elements, Refs. 5–7. By using the parabolic approximation of the dielectric constant in the cross-section this method enables us to reduce the problem of the Green's function determination for Eq. (1) to the problem of calculation of complex trajectories from the solution of an ordinary second-order differential equation.

In the case when imaginary part of the dielectric constant is equal to zero, the problem is essentially simplified. Then the radiation trajectories are real and have a clear physical meaning. The type of the Green's function for a given case can be found, for example, in Ref. 8.

In the general case, when the dielectric constant of the medium satisfies the relations (7), the Green's function is of the form

$$G(z, \rho, 0, \rho_0) = \frac{k}{2\pi i z} \frac{z_r + i z_i}{\mathrm{sh}(z_r)\cos(z_i) + i\,\mathrm{ch}(z_r)\sin(z_i)} \times \exp\left\{-\frac{k(\rho^2 + \rho_0^2)}{2}[A_r - i A_i] + k \rho \rho_0 [B_r - i B_i]\right\}, \quad (12)$$

$$A_r = F_x \frac{z_i\,\mathrm{ch}(z_r)\,\mathrm{sh}(z_r) - z_r \cos(z_i)\sin(z_i)}{\mathrm{sh}^2(z_r) + \sin^2(z_i)},$$

$$A_i = F_x \frac{z_r\,\mathrm{ch}(z_r)\,\mathrm{sh}(z_r) + z_i \cos(z_i)\sin(z_i)}{\mathrm{sh}^2(z_r) + \sin^2(z_i)},$$

$$B_r = F_x \frac{z_i\,\mathrm{sh}(z_r)\cos(z_i) - z_r\,\mathrm{ch}(z_r)\sin(z_i)}{\mathrm{sh}^2(z_r) + \sin^2(z_i)},$$

$$B_i = F_x \frac{z_r\,\mathrm{sh}(z_r)\cos(z_i) + z_i\,\mathrm{ch}(z_r)\sin(z_i)}{\mathrm{sh}^2(z_r) + \sin^2(z_i)},$$

$$z_r = \bar z \sqrt[4]{1 + \eta^{-2}} \cos(\varphi/2),$$

$$z_i = \bar z \sqrt[4]{1 + \eta^{-2}} \sin(\varphi/2), \quad \bar z = z/L_\mathrm{R},$$

$$F_x = L_\mathrm{D}/z, \quad L_\mathrm{D} = ka^2,$$

$$\eta = k\,\varepsilon_0/g_0 = L_\mathrm{D}/(g_0 L_\mathrm{R}^2), \quad \varphi = \arctan(\eta^{-1}).$$

It should be noted that this solution was obtained without using any approximations and was the exact solution of Eq. (1). Then, using Eqs. (8)–(10), we can write an expression for the intensity distribution and coherence function in the output plane, satisfying the exact equation (2)

$$W(z, \mathbf{R}) = \frac{W_{\delta\,\mathrm{ef}}}{8\pi} \frac{g_0\, z_{\mathrm{ef}} \exp(g_0 z)}{\bar{a}_0^{-2} + A_r} \frac{a^2}{z^2} \times$$

$$\times \frac{z_r^2 + z_i^2}{\mathrm{sh}(z_r) + \sin(z_i)} \exp\left(-\frac{\mathbf{R}^2}{a_{\mathrm{ws}}^2}\right),$$

$$\Gamma_2(z, \mathbf{R}{=}0, \rho) = \frac{W_{\delta\,\mathrm{ef}}}{8\pi} \frac{g_0\, z_{\mathrm{ef}} \exp(g_0 z)}{\bar{a}_0^{-2} + A_r} \frac{a^2}{z^2} \times$$

$$\times \frac{z_r^2 + z_i^2}{\mathrm{sh}(z_r) + \sin(z_i)} \exp\left(-\frac{\rho^2}{4\, a_{\rho\mathrm{s}}^2}\right), \quad (13)$$

$$a_{\mathrm{ws}}^2 = a^2\, (\bar{a}_0^{-2} + A_r)/(\bar{a}_0^{-2} A_r + A_r^2 - B_r^2),$$

$$a_{\rho\mathrm{s}}^2 = a^2\, (\bar{a}_0^{-2} + A_r)/(\bar{a}_0^{-2} A_r + A_r^2 + B_i^2),$$

$$\mu(\rho) = \frac{|\Gamma_2(z, \mathbf{R}=0, \rho)|}{\sqrt{W(z, \mathbf{R}=\rho/2)\, W(z, \mathbf{R}=-\rho/2)}} = \exp\left(-\frac{\rho^2}{4\, a_\mu^2}\right),$$

$$a_{\mathrm{m}}^2 = a^2\, (\bar{a}_0^{-2} + A_r)/(B_r^2 + B_i^2).$$

It is easy to show that, when z_i tends to zero, the solution of exact equation (2) tends to the solution of an approximate equation (4). The condition $z_i \ll 1$ is identical to the condition

$$z \ll 2\, L_{\mathrm{D}}/(g_0\, L_{\mathrm{R}}), \quad (14)$$

which determines the limits of applicability of the approximate equation. We calculated the values of the output radiation coherence radii for different dimensionless parameters of the problem characteristic of x-ray lasers. The results were obtained based on the exact and approximate solutions. Comparison of the results shows that the values of radii differ by no more than 5% if

$$z \le L_{\mathrm{D}}/(g_0\, L_{\mathrm{R}}),$$

which holds for most x-ray lasers.

It should be noted that such a coincidence is achieved when determining the coherence radius from solution of Eq. (11) with the use of Eq. (5). From the exact formula (6) it follows that for the coherence radius for solution of Eq. (11) we can write as

$$a_\mu^{-2} = a_\rho^{-2} - a_\omega^{-2}.$$

This expression not only corresponds to an exact value of a_μ, following from the solution of Eq. (13) but can lose physical meaning at large z since in this case $a_\rho > a_w$ and a square of the coherence radius a_μ becomes negative.

This fact can be explained by analyzing the integral solution of Eq. (2). It is evident (see Ref. 9) that Eq. (1) is identical to the set of equations

$$\frac{d^2\mathbf{R}}{dz^2} = \frac{1}{2} \nabla_\perp \varepsilon(z, \mathbf{R}) + \frac{1}{2k^2} \nabla_\perp (A^{-1} \Delta_\perp A(z, \mathbf{R})), \quad (15.1)$$

$$\frac{\partial A^2}{\partial z} + \nabla_\perp (\theta\, A^2) = -\, g(z, \mathbf{R}), \quad (15.2)$$

where

$$E(z, \mathbf{R}) = A(z, \mathbf{R}) \exp[iS(z, \mathbf{R})];$$

$$\theta = k^{-1} \nabla_\perp S; \quad d\mathbf{R}/dz = \theta.$$

From Eq. (15) it follows that Green's function (1) can be represented as

$$G(z, \mathbf{R}; 0, \mathbf{R}_0) \equiv A_{\mathrm{G}} \exp(iS_{\mathrm{G}}) =$$

$$= A_{\mathrm{G}} \exp\left\{ i\, kz + \frac{ik}{2} \int_0^z dz' \,[\theta^2 + \varepsilon + k^{-2} A_{\mathrm{G}}^{-1} \Delta_\perp A_{\mathrm{G}}] \right\}, \quad (16.1)$$

$$A_{\mathrm{G}} \equiv k \left| \frac{d\,\mathbf{R}(z)}{d\theta_0} \right|^{-1/2} e^{\tau/2} = k \left| \frac{d\,\mathbf{R}(z)}{d\theta_0} \right|^{-1/2} \times$$

$$\times \exp\left\{ -\frac{k}{2} \int_0^z dz'\, s(z', \mathbf{R}(z')) \right\}, \quad (16.2)$$

where A_{G} and S_{G} are the amplitude and the phase of the wave from a point source. The amplitude variation is determined by the variation of an area of an elementary ray cone, originating from a point $\{z = 0, R_0\}$, and the value of the optical depth of the amplification, τ, along these rays. Integration in Eqs. (16) is performed along the rays, whose paths satisfy Eq. (15.1). The set of Eqs. (15) is analogous to equations of geometrical optics. But the presence in the right-hand side of Eq. (15.1) of the last term results in the fact that the ray paths and phase shift along those are determined with the account for diffraction effects. And this, in its turn, has led to the assumption that the solution of this set of equations has no caustic features. If the Green's function of the medium is known, we can write the expression for the field coherence function

$$\Gamma_2(z, \rho_1, \rho_2) = \int_{-\infty}^{\infty} d\, z_{01} \int_{-\infty}^{\infty} d\, z_{02} \int_{-\infty}^{\infty} d\, \rho_{01} \int_{-\infty}^{\infty} d\, \rho_{02} < P_{\mathrm{sp}}(z_{01}, \rho_{01})\, P^*_{\mathrm{sp}}(z_{02}, \rho_{02}) > G_1\, G_2^* =$$

$$= \int_{-\infty}^{\infty} d\, \rho_0\, W_{\delta\,\mathrm{ef}}(\rho_0)\, k^2 \left| \frac{d\,\rho_1(z)}{d\theta_0} \right|^{-1/2} \left| \frac{d\,\rho_2(z)}{d\theta_0} \right|^{-1/2} \exp\{ i\, [S(\rho_1) - S(\rho_2)] + [\tau(\rho_1) + \tau(\rho_2)]/2 \}. \quad (17)$$

When deriving Eq. (17) the conditions (8) and (9) were used. Further, going to the integrated and difference coordinates and performing the Taylor expansions of the functions S and τ, the following expression is derived:

$$\Gamma_2(z, \mathbf{R}, \rho) = k^2 \left|\frac{\mathrm{d}\,\mathbf{R}(z)}{\mathrm{d}\theta_0}\right|^{-1} \times$$

$$\times \int_{-\infty}^{\infty} \mathrm{d}\rho_0\, W_{\delta\ \mathrm{ef}}(\rho_0) \exp\{i\rho\nabla_\perp S + \tau\} \exp\{(\rho\,\nabla_\perp)^2\,\tau\}. \quad (18)$$

When deriving Eq. (18) the following approximation was used:

$$\left|\frac{\mathrm{d}\,\rho_1(z)}{\mathrm{d}\theta_0}\right|^{-1/2} \left|\frac{\mathrm{d}\,\rho_2(z)}{\mathrm{d}\theta_0}\right|^{-1/2} \approx \left|\frac{\mathrm{d}\,\mathbf{R}(z)}{\mathrm{d}\theta_0}\right|^{-1},$$

$\mathbf{R} = (\rho_1 + \rho_2)/2$.

Integral solution to Eq. (4) similar to Eq. (18) can be derived as follows.

When making the Fourier transform of Eq. (4) over the difference coordinate ρ we obtain the radiation transfer equation:

$$\frac{\partial J}{\partial z} + [\boldsymbol{\theta}\,\nabla_{\mathbf{R}} + \frac{k}{2}\nabla_{\mathbf{R}}\,\varepsilon\,\nabla_{\boldsymbol{\theta}} + k\,\sigma(z, \mathbf{R})]\,J(z, \mathbf{R}, \boldsymbol{\theta}) =$$

$$= \frac{g_0}{8\pi^2 k^2}\, W_{\mathrm{ef}}\,(z, \mathbf{R}). \quad (19)$$

Its general integral solution may be presented as[3]

$$\Gamma_2(z, \mathbf{R}, \rho) = k^2 \left|\frac{\mathrm{d}\,\boldsymbol{\theta}(z)}{\mathrm{d}\mathbf{R}_0}\right| \int_{-\infty}^{\infty} \mathrm{d}\,\mathbf{R}_0\, W_{\delta\ \mathrm{ef}}(\mathbf{R}_0) \times$$

$$\times \exp\{i\,k\,\boldsymbol{\theta}(\mathbf{R}_0, \mathbf{R}, z)\,\rho + \tau(\mathbf{R}_0, \mathbf{R}, z)\}, \quad (20)$$

where the characteristics $\mathbf{R} = \mathbf{R}(z)$ obey the equation

$$\frac{\mathrm{d}^2\mathbf{R}}{\mathrm{d}z^2} = \frac{1}{2}\nabla_{\mathrm{R}}\,\varepsilon\,[z, \mathbf{R}(z)] \quad (21)$$

and the initial conditions $\mathbf{R}(z = 0) = \mathbf{R}_0$, $\mathrm{d}\mathbf{R}(z = 0)/\mathrm{d}z = \boldsymbol{\theta}_0$.

If the last term in Eq. (15.1) is omitted, it becomes identical to Eq. (21). Furthermore, if we take into account that

$$\left|\frac{\mathrm{d}\,\boldsymbol{\theta}(z)}{\mathrm{d}\mathbf{R}_0}\right| = \left|\frac{\mathrm{d}\,\mathbf{R}(z)}{\mathrm{d}\theta_0}\right|^{-1} \text{ and } \nabla_\perp S = k\,\boldsymbol{\theta}(\mathbf{R}_0, \mathbf{R}, z), \quad (22)$$

the following conclusion may be drawn. Equations (18) and (20) differ only by the exponential multiplier in Eq. (18). The reason is that in deriving Eq. (4) the corresponding component in Eq. (3) was omitted. However, we are able to verify that the use of the definition (5) for solving (20) gives the same result as the definition (6) for solving (21), i.e., with the consideration for this fact the solution (20) is similar to expression (18). The Eq. (18) follows from the exact solution (17) with the use of square approximation for the functions S and τ by the difference argument ρ and ignoring the last term in (15.1). The applicability of square approximation is determined by fulfilling the condition[3]

$\rho_c < a$,

where ρ_c is the radiation coherence radius.

As to the error, due to neglect of the last term in Eq. (15.1), earlier[3] this error was explained by the fact that in this case the diffraction effects were neglected, due to additional distortion of the beam shape at inhomogeneous amplification. However, as follows from the analysis of the set of equations (15), this term, for inhomogeneously amplifying (or absorbing) media, describes not only the diffraction effects but also the beam refraction at the inhomogeneous amplification profile. This follows from the fact that if in the set (15) $k \to \infty$, i.e., one changes to the geometrical optics approximation, this term does not become zero, and Eq. (15.1) takes the form

$$\frac{\mathrm{d}^2\mathbf{R}}{\mathrm{d}z^2} = \frac{1}{2}\nabla_\perp \{\varepsilon\,(z, \mathbf{R}(z)) + \frac{1}{4}[\int_0^z \mathrm{d}z'\,\nabla_\perp\,\sigma(z', \mathbf{R}(z'))]^2\}. \quad (23)$$

The appearance of the second term in Eq. (23) is caused by the fact that the exponent in (16.2) is proportional to k. For the media with homogeneous amplification this term vanishes since $\nabla_\perp\sigma \equiv 0$. From Eq. (23) it follows that for the media where there is no disturbance of the dielectric constant ε while inhomogeneous amplification (absorption) is available, the geometric beams deviate from rectilinear propagation and, hence, the beam undergoes refractive distortions. A characteristic length at which such distortions occur, for the media with $\varepsilon \equiv 0$ (i.e., $\eta \equiv 0$) is

$$L_\sigma = a/(\sigma_0/2)^{1/2}.$$

For active medium of an x-ray laser the condition $\eta >> 1$ is typical. In this case the refraction is determined mainly by perturbations of the dielectric constants and the amplification inhomogeneity contributes significantly to the refraction at the distance L_σ, being much greater than L_{R}

$$L_\sigma = 2\,L_{\mathrm{R}}\,\eta = 2\,L_{\mathrm{D}}/(g_0\,L_{\mathrm{R}}). \quad (24)$$

Thus the applicability of the radiation transfer equation is limited by the situations when the contribution of inhomogeneous amplification to the beam refraction is insignificant, i.e., when

$$z \ll L_{\sigma},$$

that for $\eta \gg 1$ coincides with expression (14). However, this conclusion was drawn on the basis of the results obtained using the conditions (8) i.e., for the case of amplification in the inhomogeneous active medium of spontaneous radiation. The situation, when the radiation with finite radius of coherence is incident on a medium, requires further investigation.

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