Formation of speckle interference patterns characterizing transversal or longitudinal displacement of a diffusely scattering surface. Part 2

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The sensitivity of a speckle interferometer to transversal or longitudinal displacements of a plane surface, diffusely scattering the light, is analyzed for the case when a positive lens is used at the stage of recording of a double-exposure specklogram. The interferometer's sensitivity is shown to depend on the curvature radius of a spherical wave of the coherent radiation illuminating the surface. The interferometer sensitivity to longitudinal displacement depends on the scale of the Fourier transform of the function, which characterizes the complex transmission (or reflection) amplitude of a scatterer. Experimental results correspond to theoretical prerequisites.

In the double-exposure recording of quasi-Fourier and Fourier holograms with a positive lens for the control for the transversal displacement of a plane surface, which diffusely scatters the light, the mechanism of formation of interference patterns in diffracting fields, as it was shown in Ref. 1, is caused by both uniform displacement of subjective speckles of the second exposure in the hologram plane relative to the identical speckles of the first exposure and their slope. At the stage of retrieval of the record, this leads to localization of interference patterns in two planes: in the hologram plane and in the Fourier plane. At the spatial filtering of the diffraction field, the interferometer's sensitivity to transversal displacement of a scatterer in the planes of localization of the interference patterns turns to be different.

In case of controlling for longitudinal displacement of the plane surface, diffusely scattering the light, on the one hand, a non-uniform displacement of subjective speckles, corresponding to the second exposure relative to the identical speckles of the first exposure due to difference in scales of Fourier transforms in the hologram plane of complex amplitudes of transmission (or reflection) of the scatterer takes place. On the other hand, the presence of slopes of subjective speckles, corresponding to the second exposure relative to the identical speckles of the second exposure, which vary by radius off the optical axis, causes the interference pattern localization in the hologram plane and Fourier plane. In spatial filtering of the diffraction field, the interferometer sensitivity to the scatterer longitudinal displacement in planes of localization of interference patterns turns to be different. Thus, there appears a necessity to elucidate peculiarities in correlation of intensity distributions of light, scattered by a surface in the initial and shifted positions of the scatterer based on distributions of field complex amplitudes in the plane of the photographic plate.[1]

In this paper, the formation of speckle interference patterns characterizing transversal or longitudinal displacements of a plane surface diffusely scattering the light is under analysis. The goal of the analysis is to determine interferometer sensitivity in the case when a positive lens is used at the stage of double-exposure recording of the specklogram.

According to Fig. 1, the matte screen *1* lying in the plane (x_1, y_1) is illuminated by the coherent radiation with the divergent spherical wave of the curvature radius R. The radiation, scattered diffusely by the screen, passes through the thin positive lens L with the focal length f and is registered by the photographic plate *2* lying in the plane (x_3, y_3) for the time of the first exposure. Before the second exposure in the case of control for transversal displacement, the matte screen is displaced in its plane, for instance, by the magnitude a in the direction to the x axis.

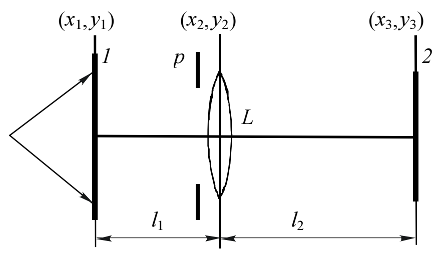


Fig. 1. Schematic of double-exposure specklogram recording: matte screen *1*; photographic plate *2*; positive lens L; aperture diaphragm p.

With allowance for diffraction restrictions,[1] distributions of complex amplitudes of fields, corresponding to the first and second expositions, in the plane of the photographic plate in the Fresnel approximation take the form

$$u_1(x_3,y_3) \sim \exp\left[\frac{ik}{2r}\left(x_3^2+y_3^2\right)\right]\times$$
$$\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)\right\}, \quad (1)$$
$$u_2(x_3,y_3) \sim \exp\left[\frac{ik}{2r}\left(x_3^2+y_3^2\right)\right]\exp\left(-\frac{ika^2}{2l}\right)\times$$
$$\times\exp\left(\frac{ikL_pax_3}{l_1l_2}\right)\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\left.\otimes\exp\left(-\frac{ikL_pax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_p}a,y_3\right)\right\}, \quad (2)$$

where $\otimes$ is the convolution; k is the wave number; l_1, l_2 are, respectively, distances between the planes (x_1, y_1), (x_2, y_2) and (x_2, y_2), (x_3, y_3); (x_2, y_2) is the principal plane of the positive lens L; L_p is the geometrical parameter of the optical system satisfying the condition $1/L_p = 1/l_1 - 1/f + 1/l_2 > 0$, i.e., $f > l_1l_2/(l_1 + l_2)$ (here $L_p < \infty$ because the condition $L_p = \infty$ corresponds to formation of a real image of the scatterer; $1/l = 1/R + 1/l_1 - L_p/l_1^2$ is a designation for brevity; $F(x_3, y_3)$ is the Fourier transform of the function $t(x_1, y_1)$, which characterizes the complex transmission amplitude of the matte screen and is a random function of coordinates, with the spatial frequencies $L_px_3/\lambda l_1l_2$, $L_py_3/\lambda l_1l_2$; λ is the wavelength of a coherent source of the light used for specklogram recording and reconstruction; $P(x_3, y_3)$ is the Fourier transform of the pupil function[2] $p(x_2, y_2)$ of the positive lens L with the spatial frequencies $x_3/\lambda l_2$, $y_3/\lambda l_2$; r is radius of curvature of the spherical wave. The value and sign of the radius are presented in Ref. 1. However, in the case of square location of the field in the plane (x_3, y_3) the exponential multiplier $\exp[ik(x_3^2 + y_3^2)/2r]$ is of no importance.

It follows from Eqs. (1) and (2) that for $\exp[-iklL_p^2(x_3^2+y_3^2)/2l_1^2l_2^2] \neq \delta(x_3, y_3)$, ($\delta(x_3, y_3)$ is the Dirac delta) a quasi-Fourier transform of the function $t(x_1, y_1)$ is formed in the plane of the photographic plate. Within the diameter[3] $D \le dl_2/L_p$, where d is the pupil diameter of the positive lens L, every point of the transform is broadened to the size of a subjective speckle, defined by the width of the function $P(x_3, y_3)$, when the diameter D_0 of the illuminated area of the matte screen satisfies the condition $D_0 \ge dl_1/L_p$, which is necessary for spatial boundedness of the scattered field. Besides, according to Eq. (2), the subjective speckles are uniformly shifted by the value al_1l_2/lL_p and sloped by the angle aL_p/l_1l_2 as compared to the distribution of the complex amplitude of the field in Eq. (1).

For the double-exposure recording of the specklogram, the distribution of its complex transmission amplitude at the linear part of the density curve of the photographic material is defined by the expression

$$\tau(x_3,y_3) \sim$$
$$\sim\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)\right\}\{c.c.\}+$$
$$+\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\left.\otimes\exp\left(-\frac{ikL_pax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_p}a,y_3\right)\right\}\{c.c.\}, \quad (3)$$

where $c.c.$ denotes the complex conjugate value.

As in Ref. 1, we take into account a particular case ($l_2 = f$), which has specific features in formation of interference patterns. For this case, distributions of complex amplitudes of fields in the plane (x_3, y_3), which correspond to the first and second exposures and distribution of the complex transmission amplitude for a double-exposure specklogram take the form

$$\tilde{u}_1(x_3,y_3) \sim \exp\left[\frac{ik}{2\tilde{r}}\left(x_3^2+y_3^2\right)\right]\times$$
$$\times\left\{\tilde{F}(x_3,y_3)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_3^2+y_3^2\right)\right]\otimes\tilde{P}(x_3,y_3)\right\}, \quad (4)$$
$$\tilde{u}_2(x_3,y_3) \sim \exp\left[\frac{ik}{2\tilde{r}}\left(x_3^2+y_3^2\right)\right]\exp\left(-\frac{ika^2}{2R}\right)\times$$
$$\times\exp\left(\frac{ikax_3}{f}\right)\left\{\tilde{F}(x_3,y_3)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\left.\otimes\exp\left(-\frac{ikax_3}{f}\right)\otimes\tilde{P}\left(x_3+\frac{f}{R}a,y_3\right)\right\}, \quad (5)$$
$$\tilde{\tau}(x_3,y_3) \sim$$
$$\sim\left\{\tilde{F}(x_3,y_3)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_3^2+y_3^2\right)\right]\otimes\tilde{P}(x_3,y_3)\right\}\{c.c.\}+$$
$$+\left\{\tilde{F}(x_3,y_3)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\left.\otimes\exp\left(-\frac{ikax_3}{f}\right)\tilde{P}\left(x_3+\frac{f}{R}a,y_3\right)\right\}\{c.c.\}, \quad (6)$$

where $\tilde{F}(x_3,y_3)$, $\tilde{P}(x_3,y_3)$ are, respectively, Fourier transforms of functions $t(x_1, y_1)$, $p(x_2, y_2)$ with spatial frequencies $x_3/\lambda f$, $y_3/\lambda f$.

It follows from Eqs. (4) and (5) that at $R \neq \infty$, a quasi-Fourier transform of the function $t(x_1, y_1)$ is formed in the plane of the photographic plate within the diameter $\tilde{D} \le d$. If the matte screen is illuminated by coherent radiation of diameter $\tilde{D}_0 \ge d$, then every point of the transform is broadened up to the size of the subjective speckle, which is defined by the width of the function $\tilde{P}(x_3,y_3)$. Besides, the subjective speckles corresponding to the second exposure are uniformly shifted by the value af/R and sloped by the angle a/f.

When retrieving the double-exposure specklogram characterizing transversal displacement of the scatterer at $l_2 \neq f$, the distribution of the field complex amplitude in the Fourier plane (x_5, y_5), according to Fig. 2 in Ref. 4 with the use of substitution of Eq. (3) into Eq. (4) from Ref. 4 and accounting for the evenness of the function $p(x_2, y_2)$, is defined by the expression

$$u(x_5,y_5) \sim \left\{ p\left(\frac{l_2}{f_0}x_5, \frac{l_2}{f_0}y_5\right) t\left(-\frac{l_1 l_2}{L_p f_0}x_5, -\frac{l_1 l_2}{L_p f_0}y_5\right) \times \right.$$
$$\times \exp\left[\frac{ikl_1^2 l_2^2}{2lL_p^2 f_0^2}\left(x_5^2 + y_5^2\right)\right] \otimes p\left(\frac{l_2}{f_0}x_5, \frac{l_2}{f_0}y_5\right) \times$$
$$\times t^*\left(\frac{l_1 l_2}{L_p f_0}x_5, \frac{l_1 l_2}{L_p f_0}y_5\right) \exp\left[-\frac{ikl_1^2 l_2^2}{2lL_p^2 f_0^2}\left(x_5^2 + y_5^2\right)\right] +$$
$$+ p\left(\frac{l_2}{f_0}x_5 + \frac{L_p}{l_1}a, \frac{l_2}{f_0}y_5\right) t\left(-\frac{l_1 l_2}{L_p f_0}x_5, -\frac{l_1 l_2}{L_p f_0}y_5\right) \times$$
$$\times \exp\left[\frac{ikl_1^2 l_2^2}{2lL_p^2 f_0^2}\left(x_5^2 + y_5^2\right)\right] \exp\left(\frac{ikl_1 l_2}{f_0 l L_p}ax_5\right) \otimes$$
$$\otimes p\left(\frac{l_2}{f_0}x_5 - \frac{L_p}{l_1}a, \frac{l_2}{f_0}y_5\right) t^*\left(\frac{l_1 l_2}{L_p f_0}x_5, \frac{l_1 l_2}{L_p f_0}y_5\right) \times$$
$$\left. \times \exp\left[-\frac{ikl_1^2 l_2^2}{2lL_p^2 f_0^2}\left(x_5^2 + y_5^2\right)\right] \exp\left(\frac{ikl_1 l_2}{f_0 l L_p}ax_5\right) \right\} \otimes$$
$$\otimes P_0(x_5,y_5), \qquad (7)$$

where f_0 is the focal distance of the positive lens L_0 [Ref. 4, Fig. 2]; $P_0(x_5, y_5)$ is the Fourier transform of the pupil function $p_0(x_4,y_4)$ of the positive lens L_0 with the spatial frequencies $x_5/\lambda f_0$, $y_5/\lambda f_0$ (the designations correspond to Ref. 4).

Based on the conclusion from Ref. 4, at a small value of the transversal displacement of the scatterer, when the value $f_0 L_p a/l_1 l_2$ is much less than the radius of the positive lens L pupil (see Fig. 1), the substitution of $p(l_2x_5/f_0 \pm L_p a/l_1, l_2y_5/f_0) \cong p(l_2x_5/f_0, l_2y_5/f_0)$ in Eq. 7 yields the visibility of the interference pattern in the Fourier plane not differing much from 1. Then, on the base of the well-known identity[5] and assumption that the period of the function $1 + \exp(ikl_1l_2ax_5/f_0lL_p)$ exceeds the size of the subjective speckle in the plane (x_5, y_5) at least by an order of magnitude,[6] the illumination distribution in the plane takes the form

$$I(x_5,y_5) \sim \left[1 + \cos\left(\frac{kl_1 l_2 a x_5}{f_0 l L_p}\right)\right] \left| p\left(\frac{l_2}{f_0}x_5, \frac{l_2}{f_0}y_5\right) \times \right.$$
$$\times t\left(-\frac{l_1 l_2}{L_p f_0}x_5, -\frac{l_1 l_2}{L_p f_0}y_5\right) \exp\left[\frac{ikl_1^2 l_2^2}{2lL_p^2 f_0^2}\left(x_5^2 + y_5^2\right)\right] \otimes$$
$$\otimes p\left(\frac{l_2}{f_0}x_5, \frac{l_2}{f_0}y_5\right) t^*\left(\frac{l_1 l_2}{L_p f_0}x_5, \frac{l_1 l_2}{L_p f_0}y_5\right) \times$$
$$\left. \times \exp\left[-\frac{ikl_1^2 l_2^2}{2lL_p^2 f_0^2}\left(x_5^2 + y_5^2\right)\right] \otimes P_0(x_5,y_5) \right|^2. \qquad (8)$$

As follows from Eq. 8, the subjective speckle structure in the Fourier plane within the area defined by the width of the function $p(l_2x_5/f_0, l_2y_5/f_0) \otimes \otimes p(l_2x_5/f_0, l_2y_5/f_0)$ has been modulated by interference fringes arranged equidistantly along the x axis. Measurement of their repetition period makes it possible to determine the value of the transversal displacement of a plane diffusely scattering the light for the known parameters λ, l_1, l_2, l, L_p, f_0. The frequency of speckle interference fringes depends on the value and sign of the curvature radius of the coherent radiation spherical wave, which is used for illumination of the scatterer at the stage of the specklogram double-exposure recording.

If $l_2 < f$, then the period $\Delta x'_5 = \lambda f_0/a(1 - l_2/f + l_1l_2/L_pR)$ of speckle interference fringes decreases with the increase of the curvature radius of the divergent spherical wavefront of the coherent radiation used for illumination of the scatterer at the stage of recording the specklogram. This is explained by the increase of displacement of its subjective speckles corresponding to the second exposure as compared with the identical speckles of the first exposure. Further, when the scatterer is illuminated by a coherent radiation with a convergent spherical wave, the period $x''_5 = \lambda f_0/a(1 - l_2/f - l_1l_2/L_pR)$ of speckle interference fringes increases with the decrease of the curvature radius R within the limits $l_1^2/(l_1 - L_p) \le R \le \infty$ up to infinity at $R = l_1^2/(l_1 - L_p)$. At the stage of recording the specklogram, the Fourier transform of the complex transmission (or reflection) amplitude of a plane surface, which diffusely scatters the light, is formed in the plane of the photographic plate.

When the Fourier transform of the function $t(x_1, y_1)$ is formed in the plane of the specklogram, there is no displacement of subjective speckles corresponding to the second exposure, just as in Ref. 7, where formation of the Fourier transform is possible only under illumination of the scatterer by coherent radiation with a convergent wavefront. Further decrease of R leads to appearance and increase of displacement of subjective speckles corresponding to the second exposure (relative to the identical speckles of the first exposure) in the plane of the specklogram. As a result, the frequency of speckle interference fringes increases.

As an example, the frequency of speckle interference fringes is presented in Fig. 2 as a function of the value and sign of R for the following fixed values: $\lambda = 0.6328$ μm, $a = 30$ μm, $f = 220$ mm, $l_1 = 350$ mm, $l_2 = 100$ mm, $f_0 = 50$ mm.

If $l_2 > f$, then the period $\Delta x'_5 = \lambda f_0/a(1 - l_2/f - l_1l_2/L_pR)$ of speckle interference fringes increases with decrease of the curvature radius of a divergent spherical wavefront of a coherent radiation used for illumination of the scatterer at the stage of recording the specklogram, within the limits $l_1^2/(l_1 - L_p) \le R \le \infty$ up to infinity as $R = l_1^2/(L_p - l_1)$.

Fulfillment of the condition $R = l_1^2/(L_p - l_1)$ corresponds to formation of the Fourier transform of $t(x_1, y_1)$ in the specklogram plane and the absence of displacement of subjective speckles of the second exposure relative to the identical speckles of the first exposure in this plane. Further decrease of R leads to appearance and increase of the displacement of subjective speckles corresponding to the second exposure in the specklogram plane. As a result, the frequency of speckle interference fringes increases. In its turn, if the scatterer is illuminated by coherent radiation with a convergent spherical wave at the stage of the double-exposure recording of the specklogram, the period $\Delta x_5'' = \lambda f_0/a(1 - l_2/f + l_1 l_2/L_p R)$ of speckle interference fringes decreases with the decrease of R. This is connected with increase of displacement of subjective speckles of the second exposure in the specklogram plane.

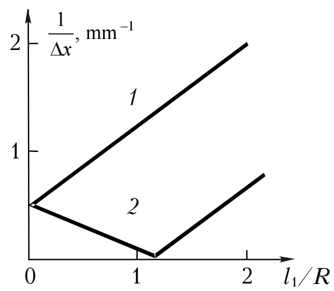


Fig. 2. Frequency of speckle interference fringes as a function of the curvature radius of a divergent spherical wavefront (*1*); of a convergent wavefront (*2*), for the case of a positive lens with f = 220 mm.

Figure 3 presents the frequency of speckle interference fringes as a function of the value and sign of R for the fixed values: $\lambda = 0.6328$ µm, $a = 30$ µm, $f = 170$ µm, $l_1 = 180$ mm, $l_2 = 300$ mm, $f_0 = 50$ mm.

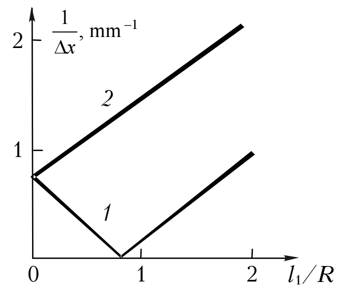


Fig. 3. Frequency of speckle interference fringes as a function of the curvature radius of a divergent spherical wavefront (*1*) and a convergent wavefront (*2*) for the case of a positive lens with f = 170 mm.

In a particular case at $l_2 = f$, when reconstructing the double-exposure specklogram characterizing transversal displacement of the plane surface diffusely scattering the light, the distribution of the complex amplitude of the field in the Fourier plane (x_5, y_5) with allowance for the substitution of Eq. (6) into Eq. (4) from Ref. 4, is defined by the expression

$$\tilde{u}(x_5,y_5) \sim \left\{ p\left(\frac{f}{f_0}x_5, \frac{f}{f_0}y_5\right) t\left(-\frac{f}{f_0}x_5, -\frac{f}{f_0}y_5\right) \times \right.$$
$$\times \exp\left[\frac{ik}{2R}\left(x_5^2 + y_5^2\right)\right] \otimes p\left(\frac{f}{f_0}x_5, \frac{f}{f_0}y_5\right) t^*\left(-\frac{f}{f_0}x_5, -\frac{f}{f_0}y_5\right) \times$$
$$\times \exp\left[-\frac{ik}{2R}\left(x_5^2 + y_5^2\right)\right] + p\left(\frac{f}{f_0}x_5 + a, \frac{f}{f_0}y_5\right) \times$$
$$\times t\left(-\frac{f}{f_0}x_5, -\frac{f}{f_0}y_5\right) \exp\left[\frac{ik}{2R}\left(x_5^2 + y_5^2\right)\right] \exp\left(\frac{ikfax_5}{f_0 R}\right) \otimes$$
$$\otimes p\left(\frac{f}{f_0}x_5 - a, \frac{f}{f_0}y_5\right) t^*\left(\frac{f}{f_0}x_5, \frac{f}{f_0}y_5\right) \exp\left[-\frac{ik}{2R}\left(x_5^2 + y_5^2\right)\right] \times$$
$$\left. \times \exp\left(\frac{ikfax_5}{f_0 R}\right) \right\} \otimes P_0(x_5,y_5), \qquad (9)$$

on the base of which the distribution of illumination in the plane takes the form

$$\tilde{I}(x_5,y_5) \sim \left[1 + \cos\left(\frac{kf}{f_0 R}ax_5\right)\right] \left| p\left(\frac{f}{f_0}x_5, \frac{f}{f_0}y_5\right) \times \right.$$
$$\times t\left(-\frac{f}{f_0}x_5, -\frac{f}{f_0}y_5\right) \exp\left[\frac{ik}{2R}\left(x_5^2 + y_5^2\right)\right] \otimes p\left(\frac{f}{f_0}x_5, \frac{f}{f_0}y_5\right) \times$$
$$\left. \times t^*\left(\frac{f}{f_0}x_5, \frac{f}{f_0}y_5\right) \exp\left[-\frac{ik}{2R}\left(x_5^2 + y_5^2\right)\right] \otimes P_0(x_5,y_5) \right|^2. \qquad (10)$$

According to Eq. (10), the period $\Delta\tilde{x}_5 = \lambda f_0 R/af$ of speckle interference fringes in the Fourier plane within the area, defined by the width of the function $p(fx_5/f_0, fy_5/f_0) \otimes p(fx_5/f_0, fy_5/f_0)$ does not depend on the sign of the curvature radius of the spherical wavefront of coherent radiation used at the stage of recording the double-exposure specklogram. The frequency of speckle interference fringes increases with the decrease of the value $|R|$ due to the increase of displacement of subjective speckles corresponding to the second exposure relative to the first exposure in the specklogram plane. The displacement is absent for $R = \infty$, when the Fourier transform of $t(x_1, y_1)$ is formed in the plane of the photographic plate at the stage of recording.[8]

Suppose that, at the stage of reconstruction of a double-exposure specklogram, characterizing transversal displacement of the scatterer, the specklogram is shifted, for instance, along the x axis by the value x_{03}. Then the distribution of the complex amplitude of the field in the plane (x_3, y_3) is defined by the expression

$$\tau(x_3,y_3) \sim \left\{ \left\{ F(x_3 + x_{03}, y_3) \otimes \right. \right.$$
$$\left. \otimes \exp\left\{-\frac{iklL_p^2}{2l_1^2 l_2^2}\left[(x_3 + x_{03})^2 + y_3^2\right]\right\} \otimes P(x_3,y_3) \right\} \{c.c.\} +$$
$$+ \left\{ F(x_3 + x_{03}, y_3) \otimes \exp\left\{-\frac{iklL_p^2}{2l_1^2 l_2^2}\left[(x_3 + x_{03})^2 + y_3^2\right]\right\} \otimes \right.$$
$$\left. \otimes \exp\left[-\frac{ikL_p a}{l_1 l_2}(x_3 + x_{03})\right] P\left(x_3 + \frac{l_1 l_2}{l L_p}a, y_3\right) \right\} \{c.c.\}. \qquad (11)$$

If we substitute Eq. (11) into the formula (4) from Ref. 4, the distribution of the complex amplitude of the field in the Fourier plane (x_5, y_5) takes the form

$$\begin{aligned} u(x_5,y_5) \sim \Bigg\{ & p\left(\frac{l_2}{f_0}x_5,\frac{l_2}{f_0}y_5\right) t\left(-\frac{l_1l_2}{L_pf_0}x_5,-\frac{l_1l_2}{L_pf_0}y_5\right)\times \\ & \times\exp\left[\frac{ikl_1^2l_2^2}{2lL_p^2f_0^2}\left(x_5^2+y_5^2\right)\right]\exp\left(\frac{i2kx_{03}x_5}{f_0}\right)\otimes p\left(\frac{l_2}{f_0}x_5,\frac{l_2}{f_0}y_5\right)\times \\ & \times t^*\left(\frac{l_1l_2}{L_pf_0}x_5,\frac{l_1l_2}{L_pf_0}y_5\right)\exp\left[-\frac{ikl_1^2l_2^2}{2lL_p^2f_0^2}\left(x_5^2+y_5^2\right)\right]\times \\ & \times\exp\left(\frac{i2kx_{03}x_5}{f_0}\right)+p\left(\frac{l_2}{f_0}x_5+\frac{L_p}{l_1}a,\frac{l_2}{f_0}y_5\right)\times \\ & \times t\left(-\frac{l_1l_2}{L_pf_0}x_5,-\frac{l_1l_2}{L_pf_0}y_5\right)\exp\left[\frac{ikl_1^2l_2^2}{2lL_p^2f_0^2}\left(x_5^2+y_5^2\right)\right]\times \\ & \times\exp\left(\frac{ikl_1l_2}{f_0lL_p}ax_5\right)\exp\left(\frac{i2kx_{03}x_5}{f_0}\right)\otimes p\left(\frac{l_2}{f_0}x_5-\frac{L_p}{l_1}a,\frac{l_2}{f_0}y_5\right)\times \\ & \times t^*\left(\frac{l_1l_2}{L_pf_0}x_5,\frac{l_1l_2}{L_pf_0}y_5\right)\exp\left[-\frac{ikl_1^2l_2^2}{2lL_p^2f_0^2}\left(x_5^2+y_5^2\right)\right]\times \\ & \times\exp\left(\frac{ikl_1l_2}{f_0lL_p}ax_5\right)\exp\left(\frac{i2kx_{03}x_5}{f_0}\right)\Bigg\}\otimes P_0(x_5,y_5), \end{aligned} \quad (12)$$

on the base of which the illumination distribution in the plane is determined by the expression

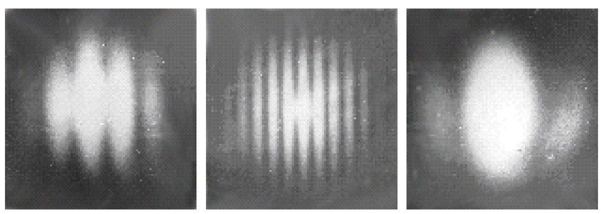
$$\begin{aligned} I(x_5,y_5) \sim & \left[1+\cos\left(\frac{kl_1l_2ax_5}{f_0lL_p}\right)\right]\Bigg| p\left(\frac{l_2}{f_0}x_5,\frac{l_2}{f_0}y_5\right)\times \\ & \times t\left(-\frac{l_1l_2}{L_pf_0}x_5,-\frac{l_1l_2}{L_pf_0}y_5\right)\exp\left[\frac{ikl_1^2l_2^2}{2lL_p^2f_0^2}\left(x_5^2+y_5^2\right)\right]\times \\ & \times\exp\left(\frac{i2kx_{03}x_5}{f_0}\right)\otimes p\left(\frac{l_2}{f_0}x_5,\frac{l_2}{f_0}y_5\right)\times \\ & \times t^*\left(\frac{l_1l_2}{L_pf_0}x_5,\frac{l_1l_2}{L_pf_0}y_5\right)\exp\left[-\frac{ikl_1^2l_2^2}{2lL_p^2f_0^2}\left(x_5^2+y_5^2\right)\right]\times \\ & \times\exp\left(\frac{i2kx_{03}x_5}{f_0}\right)\otimes P_0(x_5,y_5)\Bigg|^2. \end{aligned} \quad (13)$$

It follows from Eq. (13) that at the stage of the speckle interference pattern reconstruction its displacement in the plane (x_3, y_3) does not lead to changes in the position of speckle interference fringes (a "frozen" interference pattern). Therefore, due to the absence of interference fringes' parallax and with allowance for the fact that the constant component of transmittance of the speckle interference pattern occupies a small domain in the space, there is no necessity to perform spatial filtration of the diffraction field in recording the speckle interference pattern, which characterizes the transversal displacement of a scatterer.

Comparison of the considered speckle interferometer and the holographic interferometer,[1] in which the control for transversal displacement of a plane surface, diffusely scattering the light, is realized by registration of the interference pattern, localized in the Fourier plane, demonstrates their similar sensitivity. This is explained by similar nature of the mechanism, by which interference patterns are formed in diffusely scattered fields. The mechanism is connected with uniform displacement of subjective speckles, corresponding to the second exposure in the plane of the hologram or speckle interference pattern. At the stage of reconstruction of the hologram or the speckle interference pattern, overlapping of identical subjective speckles of two exposures is realized in the Fourier plane. The difference is that in the holographic interferometer[1] an interference pattern is recorded with spatial filtration of the diffraction field in the hologram plane, while the recording of a speckle interference pattern does not require the spatial filtration of the diffraction field. Besides, in the speckle interferometer, the threshold of the sensitivity to displacement of a scatterer is lower due to spatial increase of the speckle interference pattern.[9]

In the experiment, the double-exposure speckle interference patterns were recorded on Mikrat-VRL photographic plates irradiated by a He-Ne laser at a wavelength of 0.6328 μm. Besides, we used a positive lens with a focal length $f = 220$ mm, pupil diameter $d = 11$ mm and a positive lens with a focal length $f = 170$ mm and $d = 25$ mm. For the first lens, the distances l_1, l_2 were 350 and 100 mm, respectively, and for the second lens 180 and 300 mm. The diameter of the illuminated area of the matte screen D_0 was 50 mm. The experimental technique of the study consisted in comparison of double-exposure speckle interference patterns recorded for a fixed transversal displacement $a = (0.03 \pm 0.002)$ mm of the matte screen. The different curvature radiuses of the spherical wave of radiation used to illuminate the scatterer ranged within $120 \le |R| \le \infty$.

As an example, figure 4 shows speckle interference patterns located in the focal plane of the lens with a focal length $f_0 = 50$ mm, pupil diameter 17 mm.



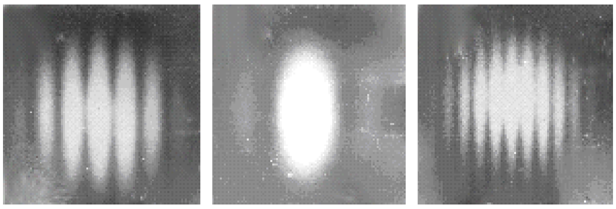
a *b* *c*

Fig. 4. Speckle interference patterns characterizing transversal displacement of a scatterer, when at the stage of recording the speckle interference pattern with a positive lens with $f = 220$ mm the matte screen was illuminated by radiation: with a plane wave (*a*); with a spherical divergent wave (*b*); with a spherical convergent wave (*c*).

Figure 4*a* corresponds to the case when the matte screen at the stage of recording the double-exposure speckle interference pattern is illuminated by a collimated beam, whereas Figure 4b corresponds to radiation with a divergent spherical wave of a curvature radius R = 350 mm, Figure 4c corresponds to a convergent spherical wave with R = 350 mm.

In these cases, like in the following ones connected with changes of the value and sign of the curvature radius, periods of interference fringes were measured (in addition to the fact that they could be determined from measurements of R for known λ, a, f, l_1, l_2, f_0). The values of the frequency of interference fringes obtained in such a way corresponded to Fig. 2 up to 10% of the allowable experimental error.

Speckle interference patterns in Fig. 5 and located in the focal plane of the lens with a focal length f_0 = 50 mm and pupil diameter 17 mm characterize the transversal displacement of the scatterer when a positive lens with a focal length f = 170 μm is used at the stage of the double-exposure recording of the speckle interference pattern. Figure 5a corresponds to the case when the matte screen is illuminated by a collimated beam, figure 5b to irradiation with a divergent spherical wave with a curvature radius R = 300 mm, figure 5c to a convergent wave with R = 300 mm.



a *b* *c*

Fig. 5. Speckle interference patterns characterizing transversal displacement of the scatterer when the matte screen is illuminated by radiation with a plane wave (a); by a spherical divergent wave (b); by a spherical convergent wave (c).

In these cases, as in the following ones, connected with the change of the value and sign of the curvature radius, periods of interference fringes were measured (in addition to the fact that they could be determined from measurements of R for known values λ, a, f, l_1, l_2, f_0). Frequencies of interference fringes obtained in such a way correspond to Fig. 3 up to an allowable experimental error (10%).

In a particular case, when a photographic plate is in the back focal plane of the lens ($l_2 = f$) at the stage of the double-exposure recording of a speckle interference pattern for the control for transversal displacement of the scatterer, the results of experimental study[10] correspond to the above-mentioned theoretical prerequisites.

Let the matte screen be displaced along the z axis by $\Delta l \ll l_1$ before the second exposure of the photographic plate *2* (see Fig. 1). Then, on the base of Ref. 1, the distribution of the complex amplitude of the field in the plane of the photographic plate for the second exposure is determined by the expression

$$u_2'(x_3, y_3) \sim \exp(ik\Delta l) \times \exp\left[\frac{ik}{2r}\left(x_3^2 + y_3^2\right)\right] \exp\left[-\frac{ikL_p^2 \Delta l}{2l_1^2 l_2^2}\left(x_3^2 + y_3^2\right)\right] \times \left\{F'(x_3, y_3) \otimes \exp\left[-\frac{ikl'L_p^2}{2(l_1 + \Delta l)^2 l_2^2}\left(x_3^2 + y_3^2\right)\right] \otimes P(x_3, y_3)\right\}, \quad (14)$$

where $F'(x_3, y_3)$ is the Fourier transform of $t(x_1, y_1)$ with the spatial frequencies $L_p' x_3 / \lambda(l_1 + \Delta l) l_2$, $L_p' y_3 / \lambda(l_1 + \Delta l) l_2$; $L_p' = L_p(1 + L_p \Delta l / l_1^2)$; $1/l' = 1/(l_1 + \Delta l) + 1/(R - \Delta l) - L_p'/(1 + \Delta l)^2$.

According to Eq. (14), the subjective speckles corresponding to the second exposure are shifted by the radius off the optical axis relative to the identical speckles of the first exposure due to the difference in scales of the Fourier transforms $F(x_3, y_3)$, $F'(x_3, y_3)$ in Eqs. (1) and (14). Moreover, this different displacement of speckles does not depend on the curvature radius of the spherical wave of coherent radiation used for illuminating the scatterer. The slope of subjective speckles is defined by the presence of the multiplier $\exp[-ikL_p^2 \Delta l(x_3^2 + y_3^2)/2l_1^2 l_2^2]$ in Eq. (14) and varies in radius off the optical axis also does not depend on the curvature radius.

Orientation character of subjective speckles, in its turn, is of the kind that there is an additional change in radius off the optical axis of their slope, which depends on the radius of curvature of the wavefront in the plane (x_1, y_1) (see Figure 1) and is defined by the multiplier standing in Eq. (14) under the convolution integral.

In general case, this leads to significant decorrelation of speckle structures of two exposures. This decorrelation is absent if the scatterer is illuminated by coherent radiation with a convergent spherical wave, whose curvature radius $R' = l_1^2/(l_1 - L_p)$, when $l_2 < f$ and the Fourier transform of $t(x_1, y_1)$ is formed in the plane of the photographic plate, or by radiation with a divergent spherical wave with the above-mentioned curvature radius R' and a quasi-Fourier transform[1] is formed in the plane of the photographic plate.

If $l_2 > f$, then the Fourier transform of $t(x_1, y_1)$ is formed in the plane of the photographic plate under illumination of the scatterer by coherent radiation with a divergent spherical wave whose curvature radius is $R' = l_1^2/(L_p - l_1)$, or a quasi-Fourier transform of $t(x_1, y_1)$ is formed in the plane of the photographic plate, irradiating by a convergent spherical wave with the above-mentioned curvature radius R'. Therefore, to prove the possibility of formation of a high-contrast speckle interference pattern, characterizing the transversal displacement of a plane surface diffusely scattering the light, we restrict ourselves by this value of the wavefront curvature radius R'. Then the distribution of the complex transmission amplitude $\tau'(x_3, y_3)$ of a double-exposure speckle interference pattern [Ref. 4, Fig. 4], ignoring the constant component of transmission, which occupies a small area in the recording plane of the speckle interference pattern, takes the form

$$\tau'(x_3,y_3) \sim \left\{F(x_3,y_3)\otimes \exp\left[-\frac{ikR'L_p^2}{4l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$

$$\left.\otimes P(x_3,y_3)\right\}\{c.c.\} + \left\{F'(x_3,y_3)\otimes\right.$$

$$\left.\otimes\exp\left[-\frac{ikR'L_p^2}{4l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)\right\}\{c.c.\}, \quad (15)$$

where spatial frequencies of the Fourier transform $F'(x_3, y_3)$ correspond to the values $L_px_3/\lambda l_1l_2(1+\Delta l/R')$, $L_py_3/\lambda l_1l_2(1+\Delta l/R')$.

Like in Ref. 4, the distribution of the complex amplitude of the field in the plane (x_5, y_5) [Ref. 4, Fig. 4] in the Fresnel approximation is determined by the expression

$$u'(x_5,y_5) \sim \iiiint \tau'(x_3,y_3)\times$$

$$\times\exp\left\{\frac{ik}{2l_3}\left[(x_3-x_4)^2+(y_3-y_4)^2\right]\right\}\times$$

$$\times p_0(x_4,y_4)\exp\left[-\frac{ik}{2f_0}\left(x_4^2+y_4^2\right)\right]\times$$

$$\times\exp\left\{\frac{ik}{2l_4}\left[(x_4-x_5)^2+(y_4-y_5)^2\right]\right\}\mathrm{d}x_3\mathrm{d}y_3\mathrm{d}x_4\mathrm{d}y_4, \quad (16)$$

which can be written[3] in the form

$$u'(x_5,y_5) \sim \exp\left[\frac{ik}{2l_4}\left(x_5^2+y_5^2\right)\right]\left\{\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2+y_5^2\right)\right]\times\right.$$

$$\times\left\{F_b(x_5,y_5)\otimes\exp\left[-\frac{ikL_b^2}{2(l_3-L_b)l_4^2}\left(x_5^2+y_5^2\right)\right]\right\}\otimes$$

$$\left.\otimes P_0(x_5,y_5)\right\}, \quad (17)$$

where L_b is the geometric parameter of the optical system recording the speckle interference pattern (it satisfies the condition $1/L_b = 1/l_3 - 1/f_0 + 1/l_4$); $F_b(x_5, y_5)$ is the Fourier transform of $\tau'(x_3, y_3)$ with the spatial frequencies $L_bx_5/\lambda l_3l_4$, $L_by_5/\lambda l_3l_4$, $P_0(x_5, y_5)$ is the Fourier transform of the pupil function $p_0(x_4, y_4)$ of the recording optical system [Ref. 4, Fig. 4] with the spatial frequencies $x_5/\lambda l_4$, $y_5/\lambda l_4$.

According to Eq. (17), a subjective speckle field is formed in the plane (x_5, y_5) with the speckle size defined by the width of $P_0(x_5, y_5)$. The phase distribution of a divergent spherical wave with the curvature radius l_4 is superposed on this field.

Substituting Eq. (15) into Eq. (17), we obtain

$$u(x_5,y_5) \sim \exp\left[\frac{ik}{2l_4}\left(x_5^2+y_5^2\right)\right]\left\{\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2+y_5^2\right)\right]\times\right.$$

$$\times\left\{\exp\left[-\frac{ikL_b^2}{2(l_3-L_b)l_4^2}\left(x_5^2+y_5^2\right)\right]\otimes\left\{p\left(\frac{l_2L_b}{l_3l_4}x_5,\frac{l_2L_b}{l_3l_4}y_5\right)\times\right.\right.$$

$$\times t\left(-\frac{l_1l_2L_b}{L_pl_3l_4}x_5,-\frac{l_1l_2L_b}{L_pl_3l_4}y_5\right)\exp\left[\frac{ikl_1^2l_2^2L_b^2}{R'L_p^2l_3^2l_4^2}\left(x_5^2+y_5^2\right)\right]\otimes$$

$$\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5,\frac{l_2L_b}{l_3l_4}y_5\right)t^*\left(\frac{l_1l_2L_b}{L_pl_3l_4}x_5,\frac{l_1l_2L_b}{L_pl_3l_4}y_5\right)\times$$

$$\times\exp\left[-\frac{ikl_1^2l_2^2L_b^2}{R'L_p^2l_3^2l_4^2}\left(x_5^2+y_5^2\right)\right]+p\left(\frac{l_2L_b}{l_3l_4}x_5,\frac{l_2L_b}{l_3l_4}y_5\right)\times$$

$$\times t\left(-\frac{l_1l_2L_b}{L_pl_3l_4}\left(1+\frac{\Delta l}{R'}\right)x_5,-\frac{l_1l_2L_b}{L_pl_3l_4}\left(1+\frac{\Delta l}{R'}\right)y_5\right)\times$$

$$\times\exp\left[\frac{ikl_1^2l_2^2L_b^2}{R'L_p^2l_3^2l_4^2}\left(x_5^2+y_5^2\right)\right]\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5,\frac{l_2L_b}{l_3l_4}y_5\right)\times$$

$$\times t^*\left(\frac{l_1l_2L_b}{L_pl_3l_4}\left(1+\frac{\Delta l}{R'}\right)x_5,\frac{l_1l_2L_b}{L_pl_3l_4}\left(1+\frac{\Delta l}{R'}\right)y_5\right)\times$$

$$\left.\left.\times\exp\left[-\frac{ikl_1^2l_2^2L_b^2}{R'L_p^2l_3^2l_4^2}\left(x_5^2+y_5^2\right)\right]\right\}\right\}\otimes P_0(x_5,y_5)\Bigg\}. \quad (18)$$

For $\Delta l \ll R'$

$$t\left[-l_1l_2L_b(1+\Delta l/R')\frac{x_5}{L_pl_3l_4},-l_1l_2L_b(1+\Delta l/R')\frac{y_5}{L_pl_3l_4}\right]=$$

$$= t\left(-l_1l_2L_b\,x_5/L_pl_3l_4,-l_1l_2L_b\,y_5/L_pl_3l_4\right)\otimes$$

$$\otimes\exp\left[-ikl_1^2l_2^2L_b^2\left(x_5^2+y_5^2\right)/2L_p^2l_3^2l_4^2\Delta l\right],$$

and taking into account that $\tau'(x_3, y_3)$ is a real function, we have

$$t^*\left[l_1l_2L_b(1+\Delta l/R')\frac{x_5}{L_pl_3l_4},l_1l_2L_b(1+\Delta l/R')\frac{y_5}{L_pl_3l_4}\right]=$$

$$= t^*\left(l_1l_2L_b\,x_5/L_pl_3l_4,l_1l_2L_b\,y_5/L_pl_3l_4\right)\otimes$$

$$\otimes\exp\left[-ikl_1^2l_2^2L_b^2\left(x_5^2+y_5^2\right)/2L_p^2l_3^2l_4^2\Delta l\right].$$

Then, as a result of integral representation of convolution with the function $\exp[-ikL_b^2(x_5^2+y_5^2)/2(l_3-L_b)l_4^2]$ in Eq. (18), the distribution of the complex amplitude of the field in the plane (x_5, y_5) takes the form

$$u'(x_5,y_5) \sim \exp\left[\frac{ik}{2l_4}\left(x_5^2+y_5^2\right)\right]\times$$

$$\times\left\{\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2+y_5^2\right)\right]\exp\left[-\frac{ikL_b^2}{2(l_3-L_b)l_4^2}\left(x_5^2+y_5^2\right)\right]\times\right.$$

$$\times\left\{\exp\left[\frac{ikL_b^2}{2(l_3-L_b)l_4^2}\left(x_5^2+y_5^2\right)\right]\otimes\left\{\left\{F_1(x_5,y_5)\otimes\right.\right.\right.$$

$$\otimes \exp\left[-\frac{ikR' L_p^2 L_b^2 l_3^2}{4 l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2}\left(x_5^2 + y_5^2\right)\right] \otimes P'(x_5, y_5)\Bigg\} \times$$

$$\times \Bigg\{ F_2(x_5, y_5) \otimes \exp\left[\frac{ikR' L_p^2 L_b^2 l_3^2}{4 l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2}\left(x_5^2 + y_5^2\right)\right] \otimes P'(x_5, y_5)\Bigg\} +$$

$$+ \Bigg\{ \exp\left[\frac{ik L_p^2 L_b^2 l_3^2 \Delta l}{2 l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2}\left(x_5^2 + y_5^2\right)\right] F_1(x_5, y_5) \otimes$$

$$\otimes \exp\left[-\frac{ikR' L_p^2 L_b^2 l_3^2}{4 l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2}\left(x_5^2 + y_5^2\right)\right] \otimes P'(x_5, y_5)\Bigg\} \times$$

$$\times \Bigg\{ \exp\left[\frac{ik L_p^2 L_b^2 l_3^2 \Delta l}{2 l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2}\left(x_5^2 + y_5^2\right)\right] F_2(x_5, y_5) \otimes$$

$$\otimes \exp\left[\frac{ikR' L_p^2 L_b^2 l_3^2}{4 l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2}\left(x_5^2 + y_5^2\right)\right] \otimes P'(x_5, y_5)\Bigg\}\Bigg\}\Bigg\} \otimes$$

$$\otimes P_0(x_5, y_5)\Bigg\}, \qquad (19)$$

where $F_1(x_5, y_5)$, $F_2(x_5, y_5)$ are, respectively, Fourier transforms of $t(l_1 l_2 L_b \xi / L_p l_3 l_4,\ l_1 l_2 L_b \eta / L_p l_3 l_4)$, $t^*(-l_1 l_2 L_b \xi / L_p l_3 l_4,\ l_1 l_2 L_b \eta / L_p l_3 l_4)$ with spatial frequencies $L_b^2 x_5 / \lambda (l_3 - L_b) l_4^2$, $L_b^2 y_5 / \lambda (l_3 - L_b) l_4^2$; $P'(x_5, y_5)$ is the Fourier transform of $p(l_2 L_b \xi / l_3 l_4,\ l_2 L_b \eta / l_3 l_4)$ with spatial frequencies $L_b^2 x_5 / \lambda (l_3 - L_b) l_4^2$, $L_b^2 y_5 / \lambda (l_3 - L_b) l_4^2$.

As $k L_p^2 L_b^2 l_3^2 \Delta l \left(x_5^2 + y_5^2\right) / 2 l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2 \ll$ $\ll k L_b^2 / 2(l_3 - L_b) l_4^2$, and the change of the function $\exp\left[ik L_p^2 L_b^2 l_3^2 \Delta l \left(x_5^2 + y_5^2\right) / 2 l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2\right]$ depending on the coordinate is small, we factor it outside the convolution integral signs in Eq. (19). Besides, we assume that the size of a subjective speckle defined by the width of $P_0(x_5, y_5)$, is less[6] than the period of $1 + \exp\left[ik L_p^2 L_b^2 l_3^2 \Delta l \left(x_5^2 + y_5^2\right) / l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2\right]$ at least by an order of magnitude. Then, taking into account the integral representation of convolution with the function $\exp\left[ik L_b^2 \left(x_5^2 + y_5^2\right) / 2(l_3 - L_b) l_4^2\right]$ in Eq. (19), the illumination distribution in the plane (x_5, y_5) is defined by the expression

$$I'(x_5, y_5) \sim \left\{1 + \cos\left[\frac{k L_p^2 L_b^2 l_3^2 \Delta l}{l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2}\left(x_5^2 + y_5^2\right)\right]\right\} \times$$

$$\times \Bigg| \exp\left[-\frac{ik L_b}{2 l_4^2}\left(x_5^2 + y_5^2\right)\right] \Bigg\{ \exp\left[-\frac{ik L_b^2}{2(l_3 - L_b) l_4^2}\left(x_5^2 + y_5^2\right)\right] \otimes$$

$$\otimes p\left(\frac{l_2 L_b}{l_3 l_4} x_5, \frac{l_2 L_b}{l_3 l_4} y_5\right) t\left(-\frac{l_1 l_2 L_b}{L_p l_3 l_4} x_5, -\frac{l_1 l_2 L_b}{L_p l_3 l_4} y_5\right) \times$$

$$\times \exp\left[\frac{ik l_1^2 l_2^2 L_b^2}{R' L_p^2 l_3^2 l_4^2}\left(x_5^2 + y_5^2\right)\right] \otimes p\left(\frac{l_2 L_b}{l_3 l_4} x_5, \frac{l_2 L_b}{l_3 l_4} y_5\right) \times$$

$$\times t^*\left(\frac{l_1 l_2 L_b}{L_p l_3 l_4} x_5, \frac{l_1 l_2 L_b}{L_p l_3 l_4} y_5\right) \exp\left[-\frac{ik l_1^2 l_2^2 L_b^2}{R' L_p^2 l_3^2 l_4^2}\left(x_5^2 + y_5^2\right)\right]\Bigg\} \otimes$$

$$\otimes P_0(x_5, y_5)\Bigg|^2. \qquad (20)$$

If the distribution of the complex amplitude of the field corresponding to the Fourier transform of $t(x_1, y_1)$ is formed at the stage of the double-exposure recording of the specklogram in the plane of the photographic plate, the illumination distribution in the plane of location of the speckle interference pattern takes the form

$$I'(x_5, y_5) \sim \left\{1 + \cos\left[\frac{k L_p^2 L_b^2 l_3^2 \Delta l}{l_1^2 l_2^2 (l_3 - L_b)^2 l_4^2}\left(x_5^2 + y_5^2\right)\right]\right\} \times$$

$$\times \Bigg| \exp\left[-\frac{ik L_b}{2 l_4^2}\left(x_5^2 + y_5^2\right)\right] \Bigg\{ \exp\left[-\frac{ik L_b^2}{2(l_3 - L_b) l_4^2}\left(x_5^2 + y_5^2\right)\right] \otimes$$

$$\otimes p\left(\frac{l_2 L_b}{l_3 l_4} x_5, \frac{l_2 L_b}{l_3 l_4} y_5\right) t\left(-\frac{l_1 l_2 L_b}{L_p l_3 l_4} x_5, -\frac{l_1 l_2 L_b}{L_p l_3 l_4} y_5\right) \otimes$$

$$\otimes p\left(\frac{l_2 L_b}{l_3 l_4} x_5, \frac{l_2 L_b}{l_3 l_4} y_5\right) t^*\left(\frac{l_1 l_2 L_b}{L_p l_3 l_4} x_5, \frac{l_1 l_2 L_b}{L_p l_3 l_4} y_5\right)\Bigg\} \otimes$$

$$\otimes P_0(x_5, y_5)\Bigg|^2. \qquad (21)$$

It follows from Eqs. (20) and (21) that the subjective speckle structure with epy speckle size defined by the width of $P_0(x_5, y_5)$, is modulated within the area, whose diameter is defined by the width of the function $p(l_2 L_b x_5 / l_3 l_4,\ l_2 L_b y_5 / l_3 l_4) \otimes$ $\otimes\, p(l_2 L_b x_5 / l_3 l_4,\ l_2 L_b y_5 / l_3 l_4)$, fringes of equal slope (the system of concentric speckle interference fringes). Measurement of their radiuses in the neighboring orders of interference provides for the determination of transversal displacement of a plane surface, diffusely scattering the light. Here, as in Ref. 4, the sensitivity of the interferometer depends on the magnitude that defines the scale of the Fourier transform of $t(x_1, y_1)$.

Besides, let us take into account that $L_b^2 l_3^2 / (l_3 -$ $- L_b)^2 l_4^2 = f_0^2 / / (l_4 - f_0)^2 = \mu^2$, where $\mu = (\tilde{l} + l_3) / l_4$ is the coefficient of scale transformation in Ref. 4. This corresponds to adequacy of the used representation of the distribution (17) of the complex amplitude of the field to that taken in Ref. 4. The conditions $\Delta l \ll R'$ and $\Delta l \ll l_1^2 l_2^2 (L_b - l_3) / 2 L_p^2 l_3^2$ imply $\tilde{l} = l_1^2 l_2^2 / 2 L_p^2 R'$. Then, to locate the speckle interference pattern at the stage of reconstruction of the specklogram in the plane, localized in the near diffraction zone, the object plane of the lens L_0 [Ref. 4, Fig. 4] must be placed at a distance $2\tilde{l}$, and the specklogram, respectively, at a distance $l_3 = \tilde{l}$ [Ref. 4].

Assume that at the stage of reconstruction of the double-exposure specklogram, characterizing the longitudinal displacement of the scatterer, the pattern is shifted, for instance, along the x axis by x_{03}. Then the distribution of the field complex amplitude in the plane (x_3, y_3) is defined by the expression

$$\tau'(x_3,y_3) \sim \left\{ F(x_3 + x_{03}, y_3) \otimes \right.$$
$$\otimes \exp\left\{ -\frac{ikR'L_p^2}{4l_1^2 l_2^2}\left[(x_3 + x_{03})^2 + y_3^2\right]\right\} \otimes P(x_3,y_3)\Bigg\}\{c.c.\} +$$
$$+\left\{ F'(x_3 + x_{03}, y_3) \otimes \exp\left\{ -\frac{ikR'L_p^2}{4l_1^2 l_2^2}\left[(x_3 + x_{03})^2 + y_3^2\right]\right\} \otimes \right.$$
$$\left. \otimes P(x_3,y_3)\right\}\{c.c.\}. \quad (22)$$

Determining the distribution of the field complex amplitude in the plane (x_5, y_5) of its location by the above-mentioned analysis, we obtain

$$u'(x_5,y_5) \sim \exp\left[\frac{ik}{2l_4}\left(x_5^2 + y_5^2\right)\right]\left\{\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2 + y_5^2\right)\right]\times\right.$$
$$\times\exp\left[-\frac{ikL_b^2}{2(l_3 - L_b)l_4^2}\left(x_5^2 + y_5^2\right)\right]\left\{\exp\left[\frac{ikL_b^2}{2(l_3 - L_b)l_4^2}\left(x_5^2 + y_5^2\right)\right]\otimes\right.$$
$$\otimes\left\{\left\{F_1\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}, y_5\right]\otimes\right.\right.$$
$$\otimes\exp\left\{-\frac{ikR'L_p^2L_b^2l_3^2}{4l_1^2l_2^2(l_3 - L_b)^2l_4^2}\left\{\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}\right]^2 + y_5^2\right\}\right\}\otimes$$
$$\otimes P'(x_5,y_5)\Bigg\}\left\{F_2\left(x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}, y_5\right)\otimes\right.$$
$$\otimes\exp\left\{-\frac{ikR'L_p^2L_b^2l_3^2}{4l_1^2l_2^2(l_3 - L_b)^2l_4^2}\left\{\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}\right]^2 + y_5^2\right\}\right\}\otimes$$
$$\otimes P'(x_5,y_5)\Bigg\} + \left\{F_1\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}, y_5\right]\times\right.$$
$$\times\exp\frac{ikL_p^2L_b^2l_3^2\Delta l}{2l_1^2l_2^2(l_3 - L_b)^2l_4^2}\left\{\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}\right]^2 + y_5^2\right\}\otimes$$
$$\otimes\exp\left\{-\frac{ikR'L_p^2L_b^2l_3^2}{4l_1^2l_2^2(l_3 - L_b)^2l_4^2}\left\{\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}\right]^2 + y_5^2\right\}\right\}\otimes$$
$$\otimes P'(x_5,y_5)\Bigg\}\left\{F_2\left(x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}, y_5\right)\times\right.$$
$$\times\exp\left\{\frac{ikL_p^2L_b^2l_3^2\Delta l}{2l_1^2l_2^2(l_3 - L_b)^2l_4^2}\left\{\left[x_5 + \frac{(l_3 - L_b)}{L_b l_3}x_{03}\right]^2 + y_5^2\right\}\otimes\right.$$
$$\otimes\exp\left\{-\frac{ikR'L_p^2L_b^2l_3^2}{4l_1^2l_2^2(l_3 - L_b)^2l_4^2}\left\{\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}\right]^2 + y_5^2\right\}\right\}\otimes$$
$$\left.\otimes P'(x_5,y_5)\right\}\Bigg\}\Bigg\}\Bigg\}\otimes P_0(x_5,y_5)\Bigg\}. \quad (23)$$

Then, basing on Eq. (23), applying the above-mentioned assumption on slow coordinate change of the function $\exp\left\{ikL_p^2L_b^2l_3^2\Delta l\left\{\left[x_5 + (l_3 - L_b)l_4x_{03}/L_b l_3\right]^2 + y_5^2\right\}/2l_1^2l_2^2(l_3 - L_b)^2l_4^2\right\}$, and taking into account the integral representation of convolution with the function $\exp\left[ikL_b^2\left(x_5^2 + y_5^2\right)/2(l_3 - L_b)l_4^2\right]$, the distribution of illumination in the plane of location of the speckle interference pattern (x_5, y_5) can be rewritten as

$$I'(x_5,y_5) \sim \left\{1 + \cos\frac{kL_p^2L_b^2l_3^2\Delta l}{l_1^2l_2^2\left(l_3 - L_b\right)^2l_4^2}\times\right.$$
$$\times\left\{\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}\right]^2 + y_5^2\right\}\Bigg\}\left|\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2 + y_5^2\right)\right]\times\right.$$
$$\times\left\{\exp\left[-\frac{ikL_b^2}{2(l_3 - L_b)l_4^2}\left(x_5^2 + y_5^2\right)\right]\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5, \frac{l_2L_b}{l_3l_4}y_5\right)\times\right.$$
$$\times t\left(-\frac{l_1l_2L_b}{L_pl_3l_4}x_5, -\frac{l_1l_2L_b}{L_pl_3l_4}y_5\right)\exp\left[\frac{ikl_1^2l_2^2L_b^2}{R'L_p^2l_3^2l_4^2}\left(x_5^2 + y_5^2\right)\right]\times$$
$$\times\exp\left(\frac{i2kL_bx_{03}x_5}{l_3l_4}\right)\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5, \frac{l_2L_b}{l_3l_4}y_5\right)\times$$
$$\times t^*\left(\frac{l_1l_2L_b}{L_pl_3l_4}x_5, \frac{l_1l_2L_b}{L_pl_3l_4}y_5\right)\exp\left[-\frac{ikl_1^2l_2^2L_b^2}{R'L_p^2l_3^2l_4^2}\left(x_5^2 + y_5^2\right)\right]\times$$
$$\left.\left.\times\exp\left(\frac{i2kL_bx_{03}x_5}{l_3l_4}\right)\right\}\otimes P_0(x_5,y_5)\right|^2. \quad (24)$$

If the distribution of the complex amplitude of the field in the plane of the photographic plate at the stage of the double-exposure recording of the specklogram corresponds to the Fourier transform of $t(x_1, y_1)$, then the distribution of illumination in the plane (x_5, y_5) is defined by the expression

$$I'(x_5,y_5) \sim \left\{1 + \cos\frac{kL_p^2L_b^2l_3^2\Delta l}{l_1^2l_2^2(l_3 - L_b)^2l_4^2}\times\right.$$
$$\times\left\{\left[x_5 + \frac{(l_3 - L_b)l_4}{L_b l_3}x_{03}\right]^2 + y_5^2\right\}\Bigg\}\left|\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2 + y_5^2\right)\right]\times\right.$$
$$\times\left\{\exp\left[-\frac{ikL_b^2}{2(l_3 - L_b)l_4^2}\left(x_5^2 + y_5^2\right)\right]\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5, \frac{l_2L_b}{l_3l_4}y_5\right)\times\right.$$

$$\times t\left(-\frac{l_1 l_2 L_b}{L_p l_3 l_4}x_5, -\frac{l_1 l_2 L_b}{L_p l_3 l_4}y_5\right)\exp\left(\frac{ikL_b x_{03} x_5}{l_3 l_4}\right)\otimes$$

$$\otimes p\left(\frac{l_2 L_b}{l_3 l_4}x_5, \frac{l_2 L_b}{l_3 l_4}y_5\right)t^*\left(\frac{l_1 l_2 L_b}{L_p l_3 l_4}x_5, \frac{l_1 l_2 L_b}{L_p l_3 l_4}y_5\right)\times$$

$$\times \exp\left(\frac{ikL_b x_{03} x_5}{l_3 l_4}\right)\Bigg\}\otimes P_0(x_5, y_5)\Bigg|^2. \quad (25)$$

According to Eqs. (24) and (25), with allowance for the inequality $l_3 < L_b$, longitudinal displacement of the specklogram at the stage of its reconstruction leads to displacement of the interference fringes in the direction opposite to the specklogram's displacement due to the parallax phenomenon.

In the particular case $l_2 = f$, at the stage of the double-exposure recording of a hologram for control over longitudinal displacement of a plane surface diffusely scattering the light, the distribution of the complex amplitude of the field corresponding to the second exposure in the object channel has the form[1]

$$\tilde{u}_2(x_3, y_3) \sim \exp(ik\Delta l)\exp\left[\frac{ik}{2\tilde{r}}\left(x_3^2 + y_3^2\right)\right]\times$$

$$\times \exp\left[-\frac{ik\Delta l}{2f^2}\left(x_3^2 + y_3^2\right)\right]\Bigg\{\tilde{F}(x_3, y_3)\otimes$$

$$\otimes \exp\left[-\frac{ik(R - \Delta l)}{2f^2}\left(x_3^2 + y_3^2\right)\right]\otimes \tilde{P}(x_3, y_3)\Bigg\}. \quad (26)$$

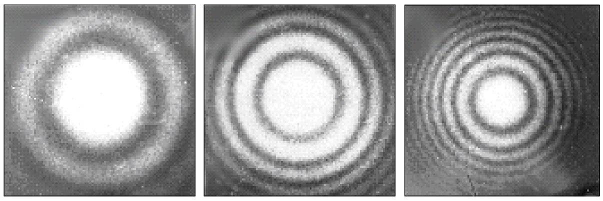
It follows from Eqs. (4) and (26) that slopes of the subjective speckles, corresponding to the second exposure, change only in radius off the optical axis in the plane (x_3, y_3) of the photographic plate, as compared with the identical speckles of the first exposure. As a result, the double-exposure recording of the specklogram is not accompanied by formation of a speckle interference pattern.

Comparison of the holographic interferometer[1] and the speckle interferometer, controlling the longitudinal displacement of a plane surface, diffusely scattering the light, demonstrates that mechanisms of interference pattern formation in these devices differ. In the holographic interferometer, the interference pattern is formed due to the change of slope angles in the hologram plane by the radius off the optical axis of the subjective speckles, corresponding to the second exposure with respect to the identical speckles of the first exposure. The formation of a speckle interference pattern in the speckle interferometer is caused by displacement of subjective speckles (varying by radius off the optical axis), corresponding to the second exposure with respect to the identical speckles of the first exposure.

Besides, spatial filtering of the diffraction field is necessary for registering interference patterns in two devices. However, in the holographic interferometer, the diameter of the filtering aperture must cover identical speckles of two exposures. In the speckle interferometer, the necessity of spatial filtration of the diffraction field is caused by two reasons: on the one hand, to reduce in the plane of the speckle interference pattern's location the area within which light intensity is concentrated due to the constant transmission component of the specklogram; on the other hand, due to parallax of interference fringes.

In the experiment, double-exposure recording of specklograms was realized with positive lenses with the above-mentioned values of f, l_1, and l_2. At the stage of specklogram recording, the diameter of the illuminated area of the scatterer was 50 mm, and the curvature radius of the wavefront R' was 212 mm, in the case of $f = 170$ mm and $R' = 533$ mm in the case of $f = 220$ mm. Values of longitudinal displacements of the matte screen were chosen in the range $\Delta l = (0.5 \pm 0.002) - (3 \pm 0{,}002)$ mm.

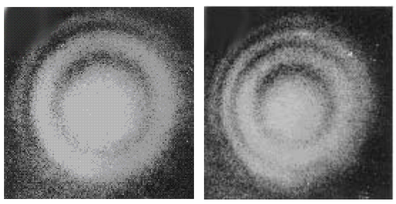
Figure 6 presents speckle interference patterns located according to Ref. 4, Fig. 4 with lens focal distance $f_0 = 50$ mm. At the stage of reconstruction of double-exposure specklograms, the diameter of the collimated beam was 2 mm, the object plane was at the distance of $(\tilde{l} + l_3) = 124$ mm, and the distance l_3 was 62 mm.



a b c

Fig. 6. Speckle interference patterns characterizing longitudinal displacement of the matte screen for the case when a positive lens with $f = 170$ mm was used at the stage of specklogram recording: $\Delta l = 0.5$ (*a*); 1 (*b*); 2 mm (*c*).

Figure 7 presents speckle interference patterns when at the stage of recording specklogram the lens focal distance $f = 220$ mm was used. In this case, the object plane was at the distance $(\tilde{l} + l_3) = 160$ mm, and the distance l_3 to the specklogram was 80 mm. In comparison with Fig. 6, the viewing angle of the lens used in the experiment did not restrict the spatial extension of the diffraction halo, within the limits of which the speckle interference pattern was observed.



a b

Fig. 7. Speckle interference patterns characterizing longitudinal displacement of the matte screen for the case when a positive lens with $f = 220$ mm was used at the stage of specklogram recording: $\Delta l = 2$ (*a*); 3 mm (*b*).

The magnitude of the longitudinal displacement of the matte screen $\Delta l = 2\lambda l_1^2 l_2^2/\mu^2 L_p^2(r_2^2 - r_1^2)$, where

r_1 and r_2 are radii of the interference fringes in the neighboring interference orders, was determined for the measured radiuses with allowance for the coefficient of scale transformation $\mu = 1.49$ in the case when the lens with $f = 170$ mm was used at the stage of specklogram recording and, respectively, $\mu = 2.2$ at $f = 220$ mm. The calculational results correspond to the presented values of Δl up to 10% of allowable experimental error. Moreover, as follows from comparison of Figs. 6 and 7, the sensitivity of the speckle interferometer with $f = 170$ mm, $l_1 = 180$ mm, and $l_2 = 300$ mm is higher due to less value of L_p/l_1l_2, which determines the scale of the Fourier transform of $t(x_1, y_1)$ in the plane of the photographic plate.

It is well-known (see, for instance, Refs. 11 and 12) that the linear part of the characteristic curve of the photographic material blackening allows a multi-exposure (as many as five exposures) recording of holograms or specklograms. Then the distribution of the complex transmission amplitude of a multi-exposure specklogram by Δl, when illumination of the scatterer at the stage of recording is realized by a spherical wave with radius of curvature, necessary to form the Fourier transform of $t(x_1, y_1)$ in the plane of the photographic plane, is determined by the expression

$$\tau'(x_3,y_3) \sim \{F(x_3,y_3) \otimes P(x_3,y_3)\mathrm{P}\}\{c.c.\} + \sum_{n=1}^{N-1}\{F'_n(x_3,y_3) \otimes P(x_3,y_3)\}\{c.c.\}, \tag{27}$$

where N is the number of expositions; $F'_n(x_3, y_3)$ is the Fourier transform of $t(x_1, y_1)$ with the spatial frequencies $L_px_3/\lambda l_1l_2(1 + n\Delta l/R')$, $L_py_3/\lambda l_1l_2(1 + n\Delta l/R')$.

Substituting Eq. (27) into Eq. (17) under the assumption that $(N - 1)\Delta l \ll R'$ and following the above-mentioned analysis of formation of the speckle interference pattern for finding the distribution of the complex amplitude of the field in the plane of its location (x_5, y_5), we obtain

$$\begin{aligned}
u'(x_5,y_5) &\sim \exp\left[\frac{ik}{2l_4}\left(x_5^2+y_5^2\right)\right]\left\{\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2+y_5^2\right)\right]\times\right.\\
&\times\exp\left[-\frac{ikL_b^2}{2(l_3-L_b)l_4^2}\left(x_5^2+y_5^2\right)\right]\left\{\exp\left[\frac{ikL_b^2}{2(l_3-L_b)l_4^2}\left(x_5^2+y_5^2\right)\right]\otimes\right.\\
&\otimes\left\{\{F_1(x_5,y_5)\otimes P'(x_5,y_5)\}\{F_2(x_5,y_5)\otimes P'(x_5,y_5)\}+\right.\\
&+\sum_{n=1}^{N-1}\left\{\exp\left[\frac{ikL_p^2L_b^2l_3^2n\Delta l}{2l_1^2l_2^2(l_3-L_b)^2l_4^2}\left(x_5^2+y_5^2\right)\right]F_1(x_5,y_5)\otimes P'(x_5,y_5)\right\}\times\\
&\times\left\{\exp\left[\frac{ikL_p^2L_b^2l_3^2n\Delta l}{2l_1^2l_2^2(l_3-L_b)^2l_4^2}\left(x_5^2+y_5^2\right)\right]F_2(x_5,y_5)\otimes\right.\\
&\left.\left.\left.\left.\otimes P'(x_5,y_5)\right\}\right\}\right\}\otimes P_0(x_5,y_5)\right\}.
\end{aligned} \tag{28}$$

Taking into account the foregoing statements and assuming that the period of the function $1+\sum_{n=1}^{N-1}\exp\left[ikL_p^2L_b^2l_3^2n\Delta l\left(x_5^2+y_5^2\right)/l_1^2l_2^2(l_3-L_b)^2l_4^2\right]$ exceeds the width of the function $P_0(x_5, y_5)$, at least by an order of magnitude, we write Eq. (28) in the form

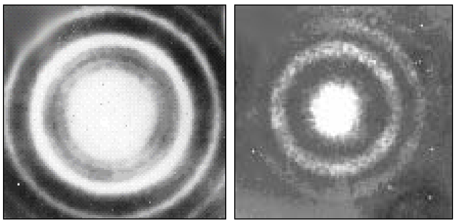
$$\begin{aligned}
u'(x_5,y_5) &\sim \exp\left[\frac{ik}{2l_4}\left(x_5^2+y_5^2\right)\right]\times\\
&\times\left\{1+\sum_{n=1}^{N-1}\exp\left[\frac{ikL_p^2L_b^2l_3^2n\Delta l}{l_1^2l_2^2(l_3-L_b)^2l_4^2}\left(x_5^2+y_5^2\right)\right]\right\}\times\\
&\times\left\{\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2+y_5^2\right)\right]\left\{\exp\left[-\frac{ikL_b^2}{2(l_3-L_b)l_4^2}\left(x_5^2+y_5^2\right)\right]\otimes\right.\right.\\
&\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5,\frac{l_2L_b}{l_3l_4}y_5\right)t\left(-\frac{l_1l_2L_b}{L_pl_3l_4}x_5,-\frac{l_1l_2L_b}{L_pl_3l_4}y_5\right)\otimes\\
&\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5,\frac{l_2L_b}{l_3l_4}y_5\right)\times\\
&\left.\left.\times t^*\left(\frac{l_1l_2L_b}{L_pl_3l_4}x_5,\frac{l_1l_2L_b}{L_pl_3l_4}y_5\right)\right\}\otimes P_0(x_5,y_5)\right\},
\end{aligned} \tag{29}$$

on the base of which and with allowance for the designation $A = kL_p^2L_b^2l_3^2\left(x_5^2+y_5^2\right)/l_1^2l_2^2(l_3-L_b)^2l_4^2$, the illumination distribution in the plane of the interference pattern location (x_5, y_5) takes a brief form

$$\begin{aligned}
I'(x_5,y_5) &\sim \left\{1+\frac{2\cos\left(\frac{N\Delta l}{2}A\right)\sin\frac{(N-1)\Delta l}{2}A}{\sin\frac{\Delta l}{2}A}+\right.\\
&\left.+\frac{\sin^2\frac{(N-1)\Delta l}{2}A}{\sin^2\frac{\Delta l}{2}A}\right\}\left|\exp\left[-\frac{ikL_b}{2l_4^2}\left(x_5^2+y_5^2\right)\right]\times\right.\\
&\times\left\{\exp\left[-\frac{ikL_b^2}{2(l_3-L_b)l_4^2}\left(x_5^2+y_5^2\right)\right]\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5,\frac{l_2L_b}{l_3l_4}y_5\right)\times\right.\\
&\times t\left(-\frac{l_1l_2L_b}{L_pl_3l_4}x_5,-\frac{l_1l_2L_b}{L_pl_3l_4}y_5\right)\otimes p\left(\frac{l_2L_b}{l_3l_4}x_5,\frac{l_2L_b}{l_3l_4}y_5\right)\times\\
&\left.\left.\times t^*\left(\frac{l_1l_2L_b}{L_pl_3l_4}x_5,\frac{l_1l_2L_b}{L_pl_3l_4}y_5\right)\right\}\otimes P_0(x_5,y_5)\right|^2.
\end{aligned} \tag{30}$$

For instance, figure 8*a* presents the speckle interference pattern corresponding to recording of five expositions with an interval of 0.5 mm when a positive lens with $f = 170$ mm is used.

At the stage of specklogram reconstruction, just as in the case of speckle interference pattern location in Fig. 6, the object plane was at the distance of 124 mm, whereas the distance from the specklogram was 62 mm. With increase of the latter by 30 mm, the illumination distribution changes and does not correspond to Eq. (30). It is presented in Fig. 8*b*.

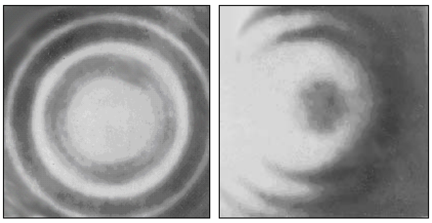


a *b*

Fig. 8. Distribution of illumination in reconstruction of a multi-exposure specklogram: in the plane of the interference pattern localization (*a*), beyond the localization plane (*b*).

Therefore, figure 8, just as experimental results,[4] shows the localization of the speckle interference pattern characterizing the longitudinal displacement of a plane surface, diffusely scattering the light in a certain plane in the near diffraction zone. Position of this plane is connected with $\tilde{l} = l_1^2 l_2^2 / 2L_p^2 R'$ and depends on the coefficient that defines the scale of the Fourier transform of the complex transmission (or reflection) amplitude of the scatterer in the specklogram plane.

Like in the case, when a negative lens is used to control for longitudinal displacement of a scatterer[4] in locating speckle interference patterns when specklograms are recorded by positive lenses, the parallax of interference fringes takes place (Fig. 9).



a *b*

Fig. 9. Speckle interference patterns demonstrating the parallax of interference fringes in non-shifted (*a*) and shifted (*b*) positions of the specklogram at the stage of its reconstruction.

Figure 9*a* corresponds to location of a speckle interference pattern presented in Fig. 8*a,* where as figure 9*b* corresponds to the case of the specklogram shifting by 5.5 mm. At such a shift, the phase of the interference pattern on the optical axis changes by π.

Thus, the theoretical analysis and experimental results demonstrate the following.

In the case of speckle interference control for the transversal displacement of a plane surface, diffusely scattering the light, when a quasi-Fourier transform of the scatterer complex transmission (or reflection) amplitude is formed in the plane of the photographic plate with the use of a positive lens at the stage of specklogram recording, the interference pattern is localized in the Fourier plane and spatial filtration of the diffraction field is not required. The sensitivity of the speckle interferometer depends on the value and sign of the curvature radius of the spherical wavefront of coherent radiation used for illumination of the scatterer, as well as on the parameter, determining the scale of the Fourier transform of the scatterer complex transmission (or reflection) amplitude. In a particular case, when the photographic plate is in the lens's back focal plane, the speckle interferometer sensitivity does not depend on the sign of the spherical wavefront curvature radius.

To provide for the speckle interference control for the longitudinal displacement of a plane surface diffusely scattering the light, it is necessary to illuminate the scatterer at the stage of specklogram recording by a coherent radiation with a spherical wavefront with the curvature radius, which satisfies the condition of formation of the Fourier transform (or quasi-Fourier transform) of the complex transmission (or reflection) amplitude of the scatterer in the plane of the photographic plate. At the stage of specklogram reconstruction, the speckle interference pattern is localized in a plane, located in the near diffraction zone, and the task of its recording requires spatial filtering of the diffraction field. Besides, sensitivity of the speckle interferometer depends on the parameter, which determines the scale of the Fourier transform of the complex transmission (or reflection) amplitude of the scatterer in the plane of the photographic plate. In a particular case, when the photographic plate is in the back focal plane of the lens, the non-uniform (varying in radius off the optical axis) displacement of subjective speckles, corresponding to the second exposure relative to the identical speckles of the first exposure, is absent and the speckle interference pattern is not formed.

References

1. V.G. Gusev, Atmos. Oceanic Opt. **19**, No. 7, 575–585 (2006).
2. J.W. Goodman, *Introduction to Fourier Optics* (McGraw–Hill, 1968).
3. V.G. Gusev, Atmos. Oceanic Opt. **5**, No. 2, 73–78 (1992).
4. V.G. Gusev, Atmos. Oceanic Opt. **20**, No. 8, 670–680 (2007).
5. V.G. Gusev, Atm. Opt. **4**, No. 5, 360–366 (1991).
6. R. Jones and C. Wykes, *Holographic and Speckle Interferometry* (Cambridge University Press, 1983).
7. V.G. Gusev, Opt. Spektrosk. **74**, No. 6, 1201–1206 (1993).
8. V.G. Gusev, Opt. Spektrosk. **69**, No. 5, 1125–1128 (1990).
9. M. Franson, *Optics of Speckles* (Mir, Moscow, 1980), 158 pp.
10. V.G. Gusev, Izv. Vyssh. Uchebn. Zaved. Ser. Fiz., No. 2, 62–70 (2006).
11. C.P. Gover, J. Opt. Soc. Am. **62**, No. 9, 1071–1077 (1972).
12. N.I. Kirillov, *High-resolving Photographic Materials for Holography and Their Processing* (Nauka, Moscow, 1979), 125 pp.