Relationship between the parameters of a scattering medium and measurement equipment in the bistatic tomographic sensing

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Relationships connecting parameters of the state of a medium under study (visibility) and parameters of the bistatic sensing scheme (the distance between receivers and sources of radiation), as well as the distance to the layers studied are obtained. It is found how the minimum spatial resolution is connected with the optical characteristics of the medium and the error of measurement equipment. The obtained equations can be helpful when choosing parameters of a bistatic tomographic sensing scheme for particular meteorological and hydrological situations.

Real-time monitoring of pollution in the environment (atmosphere, water media) is most successful when conducted from onboard an airplane (for the atmosphere) or helicopter (for a water medium). Tomographic sensing methods are preferable in this case, because they, unlike traditional schemes of remote laser sensing, require no additional *a priori* information about the functional relationship between optical characteristics (scattering and backscattering coefficients) and their spatial structure.

However, the central point of the proposed schemes of tomographic sensing[1–3] is calculation of the logarithmic derivatives of scattering signals. Taking into account that the actual experimental information usually has a discrete character and numerical differentiation of the recorded signals is an ill-posed problem, we can believe that practical realization of these methods is inefficient because even small measurement errors may lead to large errors in the optical characteristics reconstructed.[4]

In Ref. 5, we considered a bistatic scheme of tomographic sensing, which does not require calculation of logarithmic derivatives of recorded signals and is stable to various interfering factors both in emitting / receiving and recording units and in the environment, as well as to the contribution of multiple scattering. It was shown that the error in determination of optical characteristics (extinction coefficient ε) depends practically only on the error in the recorded signals. However, it seems apparently urgent to consider the relationship between the parameters of a scattering medium that characterize its state (for example, meteorological visibility range $S_{\rm m} = 3.9 / \varepsilon$), the spatial resolution, and parameters of the measuring system (accuracy, size, etc.), since the environment can take various states, while the size of carriers of the measurement equipment is limited.

For the considered bistatic sensing scheme,[5] let us first derive the equation connecting the spatial resolution L with the instrumental error and the state of the environment (cloudless atmosphere, haze, fog, cloud, water medium, etc.) characterized by the meteorological visibility range $S_{\rm m}$. The spatial resolution L here is the length of the sensing path inside the studied volume, at which the difference in the recorded signals is sufficient for these signals to be considered as different, that is, exceeding the measurement error. It is assumed that the absolute measurement error of backscattered signals Δ cannot exceed the half-difference of the recorded signals $S(R_i)$ for two positions of the sensing pulse ($\Delta < \Delta S / 2$), where $\Delta = S(R_i) - S_{\rm tr}$, $S_{\rm tr}$ is the true value of a signal, $\Delta S = S(R_i) - S(R_{i+1})$. This inequality is the condition of the maximum permissible error of signal measurement.

The recorded signals $S(R)$ are described by the optical radar equation, having the following form:

$$S(R_i, \mathbf{r}_j, R_k) = A_i\, A_k\, \beta\varphi_l(\mathbf{r}_j)\; T(R_i, \mathbf{r}_j)\; T(R_k, \mathbf{r}_j),$$

where

$$T(R_i, \mathbf{r}_j) = \exp\left\{ - \int\limits_{R_i}^{r_j} \varepsilon(\mathbf{r})\, d\mathbf{r} \right\},$$

$$T(R_k, \mathbf{r}_j) = \exp\left\{ - \int\limits_{R_k}^{r_j} \varepsilon(\mathbf{r})\, d\mathbf{r} \right\};$$

A_i and A_k are, respectively, the instrumental constants of sources and receivers ($i = 1, 2$; $k = 3, 4$); r_j are the points the scattered signal comes from ($j = 1, 2, 3, 4$); $\beta\varphi_l(r_j)$ are the coefficients of scattering at the angle φ_l at the point r_j ($l = 1, 2$). The fact that the index l takes two values indicates that we consider a particular case of symmetric sensing scheme, for which $\varphi_1 = \varphi_2$, since this case reflects all the regularities of the relationship of analyzed parameters that are inherent in the general scheme and simplifies the analysis.

Assuming that the signals $S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3)$ and $S(\mathbf{R}_2, \mathbf{r}_4, \mathbf{R}_4)$ coming to the receiver from the points $\mathbf{r}_1$ and $\mathbf{r}_4$ correspond to the ith reference level and the signals $S(\mathbf{R}_1, \mathbf{r}_3, \mathbf{R}_4)$ and $S(\mathbf{R}_2, \mathbf{r}_2, \mathbf{R}_3)$ coming from the points $\mathbf{r}_3$ and $\mathbf{r}_2$, correspond to the $(i + 1)$th level, the signal difference ΔS can be represented as follows:

$$\Delta S = S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3) - S(\mathbf{R}_2, \mathbf{r}_2, \mathbf{R}_3). \quad (1)$$

The absolute error of signal measurement $\Delta = \delta S_i\, S(R_i)$, where δS_i is the error of measurement of the signal $S(R_i)$, equals

$$\Delta = \delta[S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3)]\, S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3). \quad (2)$$

Substituting Eqs. (1) and (2) into the inequality $\Delta < \Delta S / 2$, we can write

$$\delta[S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3)]\, S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3) < 0.5\,[S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3) - S(\mathbf{R}_2, \mathbf{r}_2, \mathbf{R}_3)],$$

or

$$2\delta[S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3)] < 1 - \frac{S(\mathbf{R}_2, \mathbf{r}_2, \mathbf{R}_3)}{S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3)}.$$

Then, taking into account the lidar equation, we obtain (assuming $T(R_2, r_4) / T(R_1, r_1) \approx 1$, what is true for almost all atmospheric situations because $[R_2, r_4]$ and $[R_1, r_1]$ are equal):

$$2\delta[S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3)] < 1 - \frac{\beta\varphi_2(\mathbf{r}_1)}{\beta\varphi_1(\mathbf{r}_1)}\, T(\mathbf{r}_1, \mathbf{r}_2)\, T(\mathbf{r}_4, \mathbf{r}_2).$$

For the close reference levels meeting the condition of quasi-homogeneity $[\beta\varphi_1(\mathbf{r}_1) \approx \beta\varphi_2(\mathbf{r}_2)]$:

$$2\delta[S(\mathbf{R}_1,\mathbf{r}_1,\mathbf{R}_3)] < 1 - \exp[-\varepsilon|\mathbf{r}_2 - \mathbf{r}_1|]\exp[-\varepsilon|\mathbf{r}_2 - \mathbf{r}_4|]$$

or

$$\exp[-\bar{\varepsilon}\, L / 2] < 1 - 2\delta[S(\mathbf{R}_1, \mathbf{r}_1, \mathbf{R}_3)], \quad (3)$$

where

$$L = |\mathbf{r}_2 - \mathbf{r}_1| + |\mathbf{r}_2 - \mathbf{r}_4| + |\mathbf{r}_3 - \mathbf{r}_4| + |\mathbf{r}_3 - \mathbf{r}_1| = 2\,[|\mathbf{r}_2 - \mathbf{r}_1| + |\mathbf{r}_2 - \mathbf{r}_4|].$$

From Eq. (3), taking into account that $S_m = 3.9 / \varepsilon$, we finally obtain

$$L_{\min} > -\frac{2\ln(1 - 2\delta S)}{3.9}\, S_m. \quad (4)$$

The obtained inequality represents the dependence between the minimum spatial resolution L, the error of signal measurement δS, and the state of the scattering medium S_m. Thus, knowing the capabilities of the measuring instrumentation, we can set the minimum spatial resolution for different states of the studied medium.

When deriving Eq. (4), it was assumed that the absolute error of signal measurement Δ does not exceed the half-difference ΔS of the signals coming from two neighboring points of the medium, that is, the error of measurement of the backscattered signals should not exceed 50%. Otherwise, a physically absurd result may be obtained (for example, the atmospheric transmittance higher than unity or the negative extinction coefficient). This relationship allows us to select the minimum path length L for sounding radiation inside the studied volume, at which the signal extinction exceeds the measurement error. At $\delta S > 50\%$ it would make no sense to apply the algorithm (4), since at such errors it is inefficient (this condition was initially taken into account in the efficiency criterion (4)).

Analysis of the results of studying the spectral transmittance and scattering phase matrices in the surface atmospheric layer reveals several qualitatively different types of the optical state of the atmosphere: clouds and fog $S_m < 1$ km, mist 1 km $\le S_m \le$ 3 km, and haze $S_m > 3$ km. The extinction coefficient ε is 0.1–0.15 m^{-1} for the surface water of the World Ocean, 0.05–0.1 m^{-1} for the deep water, and up to 3.5 m^{-1} for shoaling water.[6]

The results of calculation of the minimum spatial resolution for different scattering media are shown in Figs. 1 and 2.

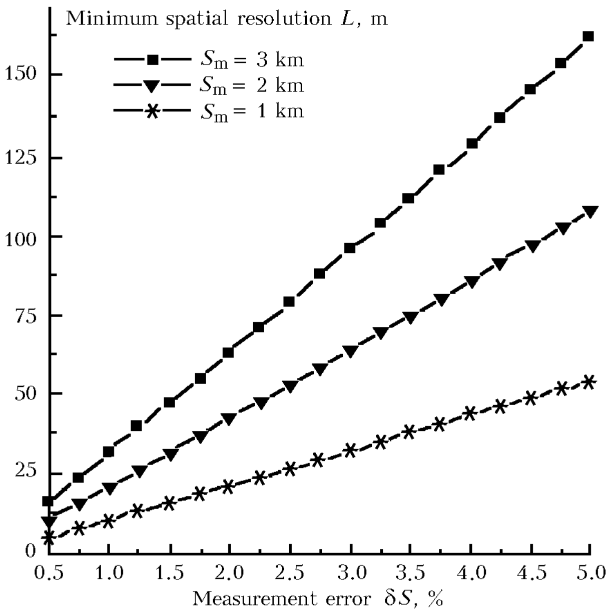

Fig. 1. Calculated minimum spatial resolution for haze.

It can be seen that L strongly depends on the measurement accuracy. If the spatial resolution of several meters is used for smokes from smoke stacks of industrial enterprises and fogs at the instrumental error δS = 2%, then for haze the resolution should be about tens of meters (see Fig. 1).

For water media, as follows from analysis of Fig. 2, the minimum spatial resolution may achieve several centimeters.

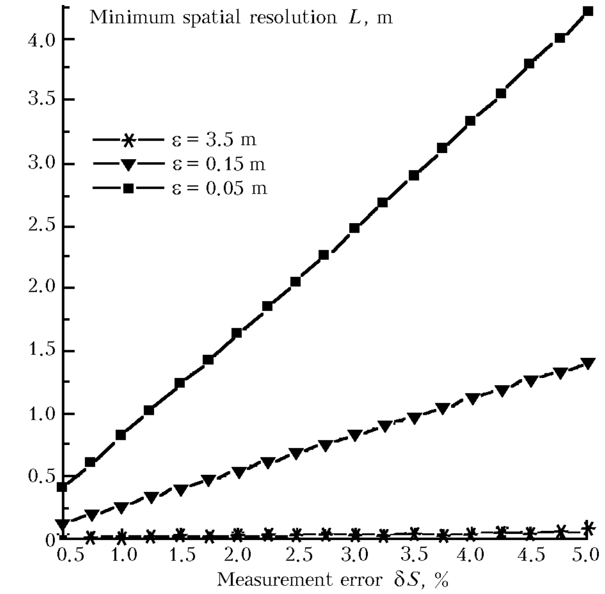

Fig. 2. Calculated minimum spatial resolution for water medium.

Let us derive now the relationship of the distance to the studied object with the size of the measurement system, measurement error, and the state of the scattering medium.

We use the geometry of the sensing scheme from Ref. 5 and the following equation:

$$L = |\mathbf{r}_2 - \mathbf{r}_1| + |\mathbf{r}_2 - \mathbf{r}_4| + |\mathbf{r}_3 - \mathbf{r}_4| + |\mathbf{r}_3 - \mathbf{r}_1|.$$

Consider a triangle with vertices at the points $\mathbf{R}_1$, $\mathbf{r}_1$, and $\mathbf{R}_3$:

$$\tan\alpha = |\mathbf{r}_1 - \mathbf{R}_3| \,/\, |\mathbf{R}_3 - \mathbf{R}_1| = d\,/\,\Delta_1, \qquad (5)$$

and a triangle with vertices at the points $\mathbf{r}_1$, $\mathbf{r}_3$, and $\mathbf{r}_4$:

$$\tan\alpha = |\mathbf{r}_3 - \mathbf{r}_4| \,/\, |\mathbf{r}_1 - \mathbf{r}_4| = |\mathbf{r}_3 - \mathbf{r}_4| \,/\, \Delta_2;$$

$$|\mathbf{r}_2 - \mathbf{r}_4| = |\mathbf{r}_3 - \mathbf{r}_1| = \frac{\Delta_2}{\cos\alpha}. \qquad (6)$$

Taking into account Eqs. (5) and (6), we can easily obtain

$$|\mathbf{r}_2 - \mathbf{r}_1| = |\mathbf{r}_3 - \mathbf{r}_4| = d\,\frac{\Delta_2}{\Delta_1}. \qquad (7)$$

Based on Eqs. (6) and (7), the equation for L has the following form:

$$L = 2\left\{d\,\frac{\Delta_2}{\Delta_1} + \frac{\Delta_2}{\cos\alpha}\right\}.$$

With allowance for Eq. (4) and the trigonometric equation $\cos(\arctan x) = \frac{1}{\sqrt{1 + x^2}}$, we finally have for the distance d to the scattering volume studied

$$d > 0.5\,\Delta_1\left(\frac{L_{\min}}{2\Delta_2} - \frac{2\Delta_2}{L_{\min}}\right). \qquad (8)$$

The obtained equation (8) allows us, for a given $L_{\min}$ with the allowance for the parameters of the sensing scheme (given distance between the receivers (Δ_2) and source and receiver (Δ_1)) to determine the distance to the studied volume, at which it is possible to find the optical characteristics of the volume with the given measurement error (δS) and the state of the medium ($S_{\rm m}$).

Let us analyze the limiting cases in Eq. (8). At $L_{\min} = 2\Delta_2$, $d = 0$. This means that the path passed by the sounding pulse is equal to the distance between the receivers. In this case, sensing pulses should apparently be emitted in the opposite direction along the straight line connecting these receivers. Then usually $\Delta_1 = 0$ as well, that is, sources and receivers should be set at the same points. The sensing scheme with sensing pulses emitted in the opposite direction was considered in Ref. 7. At $\Delta_2 \to 0$ (close receivers) $d \to \infty$. This means that the given value of $L_{\min}$ will be achieved in the positive half-plane at infinity. If $L \to 0$, then $d \to -\infty$. The physical meaning of this result is that the given small value $L_{\min} \to 0$ in the positive half-plane will be achieved with the sensing pulse emitted from the negative half-plane. It can be concluded from the above-said that to obtain $L_{\min} = 2\Delta_2$ at a given Δ_2, we cannot consider arbitrarily small values of $L_{\min}$ (less than Δ_2).

Analysis of the results on d calculated by Eq. (8) that are given in Table 1 suggests that to obtain the correct results (assuming fulfillment of Eq. (4)) at the given parameters of the sensing scheme (Δ_2, Δ_1) and the state of the object under study ($S_{\rm m}$), as the measurement error δS increases, the distance d to the studied volume should be increased.

For water scattering media ($\varepsilon \sim 0.01$–$1\ {\rm m}^{-1}$), the values of d at Δ_2 equal to units or tens of centimeters are much smaller, since $S_{\rm m}$ and, correspondingly, $L_{\min}$ for such media are two to three orders of magnitude smaller than those in the atmosphere.

The following question may arise: what separation between the source and receiver (Δ_1) is needed to measure optical characteristics of the given volume ($S_{\rm m}$) being at the distance d with the given measurement error δS at the separation Δ_2 between the receivers? To solve this problem, we use Eq. (8). After transformations, the solution for Δ_1 takes the following form:

$$\Delta_1 \le 4\,\frac{\Delta_2\,L_{\min}}{L_{\min}^2 - 4\Delta_2^2}\,d. \qquad (9)$$

This means that to measure the characteristics of a volume being at the distance d with the given error (δS) and the separation between the receivers (Δ_2), the maximum separation Δ_1 between the radiation sources and receivers can be determined by Eq. (9). It should be noted that Eq. (9) is valid only under the condition $2\Delta_2 \le L_{\min}$. Otherwise (at $L_{\min} < 2\Delta_2$), no limits are imposed on Δ_1.

It follows from Eq. (9) that at $2\Delta_2 = L_{\min}$ (sensing pulses emitted along the opposite directions) the distance Δ_1 between the emitting / receiving devices is principally unlimited ($\Delta_1 \to \infty$), although in practice, because of the finite parameters of the emitting / receiving devices (energy, sensitivity, etc.) and the presence of background radiation and noise, Δ_1 cannot be infinitely long. Values of Δ_1 calculated by Eq. (9) are given in Table 2.

Table 1. Calculated distance to the object of study at the given parameters of the sensing scheme, measurement error, and the state of the medium

δS = 0.5%, S_m = 20 km						
	Δ_1 = 1 m			Δ_1 = 5 m		
	Δ_2 = 1 m	Δ_2 = 3 m	Δ_2 = 5 m	Δ_2 = 3 m	Δ_2 = 5 m	Δ_2 = 10 m
$d \geq$	25.8 m	8.6 m	5.1 m	42.8 m	25.5 m	12.4 m
δS = 3%, S_m = 20 km						
	Δ_1 = 1 m			Δ_1 = 5 m		
	Δ_2 = 1 m	Δ_2 = 3 m	Δ_2 = 5 m	Δ_2 = 3 m	Δ_2 = 5 m	Δ_2 = 10 m
$d \geq$	158.7 m	52.9 m	31.7 m	264.4 m	158.6 m	79.2 m
δS = 0.5%, ε = 0.05 m$^{\$1}$						
	Δ_2 = 0.1 m			Δ_2 = 0.2 m		
	Δ_1 = 0.1 m	Δ_1 = 0.3 m	Δ_1 = 0.5 m	Δ_1 = 0.1 m	Δ_1 = 0.3 m	Δ_1 = 0.5 m
$d \geq$	0.08 m	0.23 m	0.38 m	0.0005 m	0.002 m	0.003 m
δS = 3%, ε = 0.05 m$^{\$1}$						
	Δ_2 = 0.1 m			Δ_2 = 0.2 m		
	Δ_1 = 0.1 m	Δ_1 = 0.3 m	Δ_1 = 0.5 m	Δ_1 = 0.1 m	Δ_1 = 0.3 m	Δ_1 = 0.5 m
$d \geq$	0.62 m	1.84 m	3.07 m	0.19 m	0.58 m	0.97 m
δS = 0.5%, ε = 0.1 m$^{\$1}$						
	Δ_2 = 0.1 m			Δ_2 = 0.05 m		
	Δ_1 = 0.1 m	Δ_1 = 0.3 m	Δ_1 = 0.5 m	Δ_1 = 0.05 m	Δ_1 = 0.1 m	Δ_1 = 0.2 m
$d \geq$	0.0005 m	0.002 m	0.003 m	0.038 m	0.076 m	0.15 m
δS = 3%, ε = 0.1 m$^{\$1}$						
	Δ_2 = 0.1 m			Δ_2 = 0.2 m		
	Δ_1 = 0.1 m	Δ_1 = 0.3 m	Δ_1 = 0.5 m	Δ_1 = 0.1 m	Δ_1 = 0.3 m	Δ_1 = 0.5 m
$d \geq$	0.30 m	0.90 m	1.51 m	0.14 m	0.42 m	0.69 m

Table 2. Calculated source / receiver separation at the given distance to the object under study, measurement error, and the state of the medium

δS = 0.5%, S_m = 39 km				
	d = 500 m		d = 3000 m	
	Δ_2 = 1 m	Δ_2 = 5 m	Δ_2 = 1 m	Δ_2 = 5 m
$\Delta_1 \leq$	9.95 m	49.87 m	59.71 m	299.23 m
δS = 3%, S_m = 39 km				
	d = 500 m		d = 3000 m	
	Δ_2 = 1 m	Δ_2 = 5 m	Δ_2 = 1m	Δ_2 = 5 m
$\Delta_1 \leq$	1.62 m	8.08 m	9.69 m	48.49 m
δS = 0.5%, S_m = 20 km				
	d = 500 m		d = 3000 m	
	Δ_2 = 1 m	Δ_2 = 5 m	Δ_2 = 1 m	Δ_2 = 5 m
$\Delta_1 \leq$	19.41 m	97.93 m	116.46 m	587.6 m
δS = 3%, S_m = 20 km				
	d = 500 m		d = 3000 m	
	Δ_2 = 1 m	Δ_2 = 5 m	Δ_2 = 1 m	Δ_2 = 5 m
$\Delta_1 \leq$	3.15 m	15.76 m	18.91 m	94.57 m

The calculated results on the separation between the sources and receivers Δ_1 by Eq. (9) at the given parameters of the sensing scheme Δ_2 and the distance to the object of sensing indicate that, on the one hand, to achieve the minimum spatial resolution with the increasing measurement error δS, the separation Δ_1 should be decreased (at S_m = const) and, on the other hand, at the constant level of the error δS with the decreasing meteorological visibility range (S_m), Δ_1 should be increased.

Thus, we have derived the relationships between the parameters of the tomographic sensing scheme (separation between the source and receiver Δ_1, separation between the receivers Δ_2), the distance to the object of sensing d, the measurement error δS, and the state of the medium under study (S_m). The obtained relationships allow one to specify the parameters of the sensing scheme in almost any experimental situation.

References

1. M.M. Kugeiko, œ*Method for determination of optical characteristics of scattering media*,B USSR Inventor's Certificate No. 1798664, ICI G01W1 / 00.
2. V.V. Veretennikov, Atm. Opt. **4**, No. 6, 462$469 (1991).
3. M.M. Kugeiko, Issled. Zemli iz Kosmosa, No. 1, 42$45 (2000).
4. A.M. Tikhonov and V.Ya. Arsenin, *Methods for Solution of Ill-Posed Problems* (Nauka, Moscow, 1979), 286 pp.
5. M.M. Kugeiko and D.M. Onoshko, Atmos. Oceanic Opt. **11**, No. 12, 1147$1149 (1998).
6. V.A. Volokhtyuk, V.M. Kochetkov, and R.R. Krasovskii, *Problems of Optical Radar* (Sov. Radio, Moscow, 1972), 256 pp.
7. V.A. Firago and M.M. Kugeiko, in: *Proceedings of the III Conference on Laser Physics and Optoelectronics* (Minsk, 1997), pp. 314$317.