THE EFFICIENT OUTER SCALE OF ATMOSPHERIC TURBULENCE

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We discuss some possibilities of introducing the distortions of an optical wave phase, propagating along vertical atmospheric path, as an integral characteristic, describing the turbulence along the path. Several models of the turbulence outer scale profile have been analyzed as well as the structural characteristic of the atmospheric refractive index fluctuations in order to find the value of the efficient outer scale. The error in the Strehl ratio determination was estimated. This ratio was computed using the efficient outer scale and compared to its value computed by the model profile of the outer scale.

1. INTRODUCTION

The work on design of a large telescope requires the knowledge of its predictable characteristics including the information about parameters of a model[1] of height profiles of atmospheric turbulence such as turbulence intensity and the outer turbulence scale in the planned location of the telescope. These characteristics are the point spread function (PSF) and the efficient angular resolution.

One of the traditional ways to estimate the angular resolution of a designed telescope is to measure the parameters of the image (long-exposition PSF) obtained on a telescope of small diameter. However, the turbulent PSF of a small telescope will correspond to the PSF of a larger telescope if only the outer turbulence scale considerably exceeds the dimension of the telescope diameter in both cases. According to some experimental works[2] performed in different observatories throughout the world in recent years, this condition is broken for modern projects of telescopes with aperture dimensions of the order of 8–10 m (VLT 4×8 m, Keck 2×10). Saying about the outer scale one should keep in mind that this parameter changes with the height, i.e., it is necessary to use the information about measured parameters of the model of height profiles of atmospheric turbulence.

The influence of the outer scale on spatial and dynamic characteristics of the phase distortions for different models of the atmospheric turbulence spectrum in the region of low frequencies (i.e., large spatial scales) was considered in several papers.[3–7] The possibility to introduce an efficient outer scale as an integral characteristic of turbulence is of great interest as it can permit one to change the height profile for the outer scale. One of the reasons to introduce this parameter is that the applicability of the models of height profiles of atmospheric turbulence is restricted due to their dependence on geographical location. It will also permit one to simplify mathematical calculations connected with the account for influence of the atmospheric turbulence on the phase of optical waves.

In this paper we consider some related problems: the principal possibility to introduce such a parameter, the class of problems in which it is worth to be applied, and the accuracy of description. We also study how the change of the height profile for the efficient outer scale influences the image parameters; in particular, we estimate the error in calculation of the Strehl parameter of a turbulent PSF by use of the efficient outer scale in comparison with the value obtained by calculation with respect to the model height profile of the outer scale.

2. PARAMETERS OF THE MODEL OF HEIGHT PROFILES OF ATMOSPHERIC TURBULENCE

Semiempirical profiles of turbulence intensity $C_n^2(h)$ corresponding to the best

$$C_n^2 = (h[\mathrm{km}]) = 5.19\cdot 10^{-16}\cdot 10^{-0.86h} + 10^{-18.34+0.29h-0.0284h^2+0.000743h^3}$$

and the worst

$$C_n^2 = (h[\mathrm{km}]) = 9.50\cdot 10^{-14}\cdot 10^{-2.09h} + 10^{-14.39+0.17h-0.0348h^2+0.000959h^3}$$

vision conditions were taken from Ref. 8.

At present, there exist a lot of models of height profiles $L_0(h)$. Some models chosen for the study are presented below:

$$(A) L_0(h) = \begin{cases} 0.4 & h \le 1 \text{ m}; \\ 0.4h & h < 1 \text{ m}; \end{cases}$$

$$(B) L_0(h) = \begin{cases} 0.4 & h \le 1 \text{ m}; \\ 0.4h & 1 < h \le 25 \text{ m}; \\ 2\sqrt{h}, & h > 25 \text{ m}; \end{cases}$$

$$(C) L_0(h) = \begin{cases} 0.4 & h \le 1 \text{ m}; \\ 0.4h & 1 < h \le 25 \text{ m}; \\ 2\sqrt{h}, & 25 < h \le 1000 \text{ m}; \\ 2\sqrt{1000}, & h > 1000 \text{ m}; \end{cases}$$

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$$(D) L_0(h) = \frac{5}{1 + \left[\frac{h - 7500}{2000}\right]^2};$$

$$(E) L_0(h) = \frac{4}{1 + \left[\frac{h - 8500}{2500}\right]^2}.$$

The model (A) is recommended in Ref. 9 for small heights, (B) is proposed by D.L. Fried,[1,10] and (C) is a generalization of (A) and (B). The model (A) was not studied in this paper because of the limitedness of its applications. The models (D) and (E) are obtained by generalizing results of measurements performed in the USA, France, and Chile.[1,11,12]

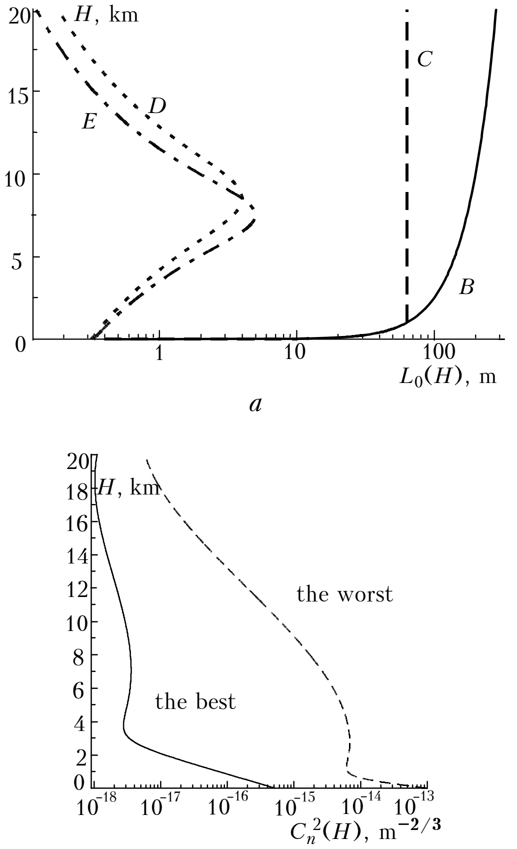

FIG. 1. The height profiles of the outer scale $L_0(h)$ (a) and turbulence intensities $C_n^2(h)$ (b).

The vertical profiles corresponding to these models are presented in Fig. 1. As one can see, the graphs (D) and (E) are similar in the character of growth and the presence of the maximal value at a certain finite height; so one can study one of the models and generalize the obtained results for the second. Here we chose the model (D). By the same criteria, it is possible to choose one model from (B) and (C), let it be the model (C).

3. THE EFFICIENT OUTER SCALE OF ATMOSPHERIC TURBULENCE

3.1. Determination Methods

We propose two methods for determining the effective outer scale, namely, by the discrepancy between structure functions of phase fluctuations and by the saturation level.

3.1.1. Determination by the discrepancy

To determine the effective outer scale by this model, minimization of the integral square discrepancy of structure functions of phase fluctuations $\Delta(L_0) = \int_0^{\rho_{max}} [D_\varphi(\rho, L_0) - D_\varphi(\rho, L_0(h))]^2 d\rho$ is used. Here $D_\varphi(\rho, L_0(h))$ is the structure function corresponding to the height profile of the outer scale $L_0(h)$, $D_\varphi(\rho, L_0)$ is the function corresponding to the constant value of the outer scale L_0. The variable ρ_{max} depends on the studied range (Fig. 2) and has a value of either 10 m (what corresponds to the largest diameter of existing telescopes) or Arg(90%), i.e., the argument at which the structure function reaches 90% of the saturation level.

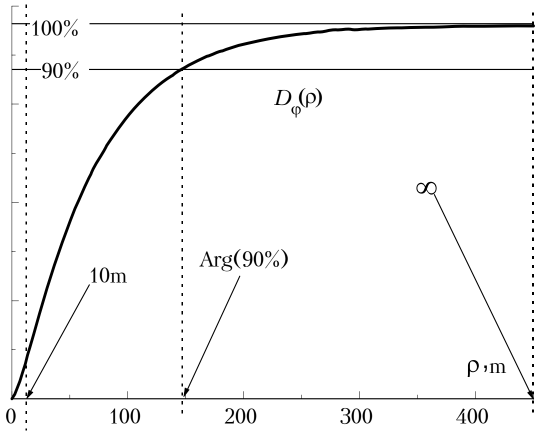

FIG. 2. Variants of introducing ρ_{max}.

The structure function was calculated by modified von Karman spectrum of atmospheric turbulence $\Phi_n(\varkappa, \xi) = 0.333 C_n^2(\xi)(\varkappa^2 + L_0^{-2}(\xi))^{-11/6}$, where ξ is the current coordinate along the propagation path; for the case of a vertical path, $\xi = h$ where h is the height over the underlying surface.

The discrepancy introduced in such a way defines the divergence degree for two structure functions. The value of the outer scale L_0^* at which the discrepancy $\Delta(L_0)$ is minimal will be called the effective outer scale of atmospheric turbulence.

3.1.2. Determination by the Saturation Level

The name of the method is directly derived from the fact that the value of argument at which the structure function saturates is taken as the upper boundary of the range studied

$$L_0^* = \left[\int\limits_0^\infty L_0^{5/3}(h) C_n^2(h) \mathrm{d}h \Big/ \int\limits_0^\infty C_n^2(h) \mathrm{d}h \right]^{3/5}.$$

The results presented below indicate that the method is close to the determination by discrepancy [0...Arg(90%)] in its characteristics.

3.2. Comparison of the Results

We shall only comment the results of applying the method [0...10 m] to the profile (*C*) since other methods and profiles have qualitatively similar results.

Figure 3*a* presents the graph of the structure function corresponding to the profile $L_0(h) - C$ together with the family of structure functions calculated using some constant values of the outer scale. One can assume that there exists certain value L_0^* at which the functions $D_\varphi(\rho, L_0^*)$ and $D_\varphi(\rho, L_0(h) - C)$ are most close. Figure 3*b* demonstrates that this assumption is true, namely, the minimum corresponding to the value $L_0^* \approx 32.5$ m is shown by the dashed line. Comparing the curves $D_\varphi(\rho, L_0(h) - C)$ and $D_\varphi(\rho, L_0^* = 32.5 \text{ m})$ in Fig. 3*c*, we see their similarity indicating the efficiency of the method for the profile $L_0(h) - C$.

The results of calculation of L_0^*(in meters) by the above-mentioned methods for different models of $L_0(h)$ and $C_n^2(h)$ are presented in Tables I and II.

TABLE I.

Model $L_0(h)$	$C_n^2(h)$ – the best		
	Method		
	0...10 m	0...Arg(90%)	0...∞
(*B*)	34.7	50.6	58.4
(*C*)	32.5	39.9	42.9
(*D*)	0.60	0.66	0.71
(*E*)	0.68	0.75	0.84

TABLE II.

Model $L_0(h)$	$C_n^2(h)$ – the worst		
	Method		
	0...10 m	0...Arg(90%)	0...∞
(*B*)	55.4	88.5	98.0
(*C*)	40.6	49.3	52.3
(*D*)	1.04	1.13	1.78
(*E*)	1.31	1.46	1.56

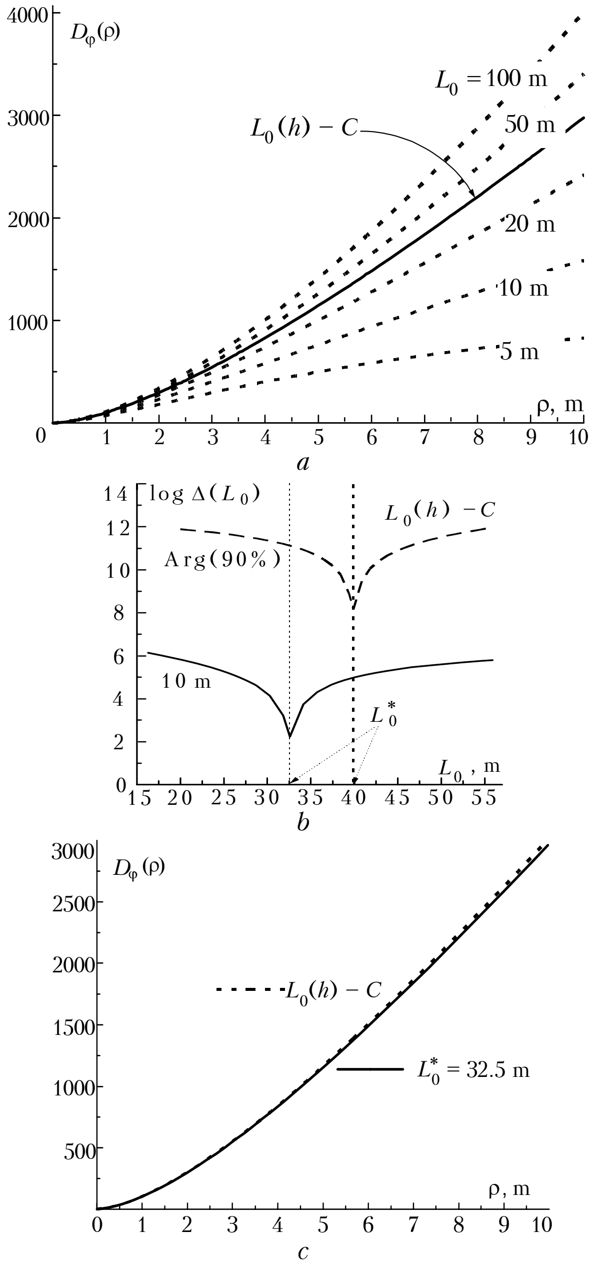

FIG. 3. The structure function for the profile (C) and the family of structure functions calculated for fixed values of L_0 (a). Discrepancy in the method [0...10 m] *and the method* [0...Arg(90%)] *(b). The structure function for the profile $L_0(h) - C$ and for the corresponding effective external scale L_0^* (c).*

3.2.1. Comparison of the methods

Analysis of the value L_0^* obtained by different methods for the same height profile $L_0(h)$ demonstrates that its growth (i.e., L_0^* [0...10 m] < < L_0^*[0...Arg(90%)] < L_0^* [0...∞]) is caused by the necessity to compensate for increasing influence of the $D_\varphi(\rho)$ portions at large argument values with the increase of ρ_{max} (i.e., ρ_{max} [0...10 m] < < ρ_{max} [0...Arg(90%)] < ρ_{max} [0...∞]). To reduce the increased discrepancy, i.e., to reduce the area between two structure functions, it is necessary "to lift" the structure function $D_\varphi(\rho,\, L_0^*)$ to the structure function $D_\varphi(\rho,\, L_0(h))$. And Fig. 3*a* demonstrates that the "lift" of $D_\varphi(\rho,\, L_0^*)$ occurs with the increase of the value L_0^*.

3.2.2. Comparison by the models of $C_n^2(h)$ used

Studying the dependence of the value L_0^* on the model of $C_n^2(h)$ one can say that lower value of L_0^* for the "best" vision conditions is caused by essential distinctions in the behavior of $C_n^2(h)$. As one can see in Fig. 1*b*, that the "best" profile $C_n^2(h)$ rapidly falls off with the growth of height, and the probability of appearance of large-scale fluctuations diminishes. This leads to the decrease of the structure function and L_0^*.

3.2.3. Comparison by the models of $L_0(H)$ used

The considerable difference between the value L_0^* for the models $L_0(h) - C$ and $L_0(h) - D$ can be explained by the following reasoning. The characteristic property of the model $L_0(h) - D$ is the presence of a finite maximal value of L_0 followed by its decrease at heights above 7–8 km (see Fig. 1*a*), so the appearance of larger scales is impossible. At the same time, the growth of L_0 with the height is inherent in the model $L_0(h) - C$ what increases the influence of large-scale fluctuations and, consequently, one can expect the growth of $D_\varphi(\rho)$ what finally leads to the increase of L_0^*.

4. STREHL PARAMETER

The turbulent broadening of an image leads to a decrease in its peak intensity. It is well-known that the level of decrease is defined by Strehl parameter SR which is the ratio of peak intensity of the turbulent image ($I_{\rm turb}$) to that of the diffraction limited image ($I_{\rm diffr}$). One can immediately determine SR using the structure function of phase fluctuations $D_\varphi(\rho)$. Indeed, it can be demonstrated that

$$SR = \frac{I_{\rm turb}}{I_{\rm diffr}} = \frac{\int\limits_0^D \rho \mathrm{d}\rho\ \tau_0(\rho) \mathrm{e}^{-(1/2)D_\varphi(\rho)}}{\int\limits_0^D \rho \mathrm{d}\rho\ \tau_0(\rho)},$$

where D is the diameter of the telescope and $\tau_0(\rho)$ is the optical transmitting function.

For a homogeneous propagation path, i.e., under the condition that L_0 does not depend on the height over the underlying surface, SR can be easily determined from the telescope diameter D, coherence radius r_0, and L_0. The calculational results for a homogeneous path are presented in Fig. 4. As one can see from the figure, SR decreases with the growth of L_0 because the increase of L_0 leads to the increase of image distortions by the atmospheric turbulence. The dependence of SR on the ratio D/r_0 shows that the increase of D/r_0 at fixed D is indicative of the decrease of the coherent part of the aperture what leads to the decrease of peak intensity of the turbulent image.

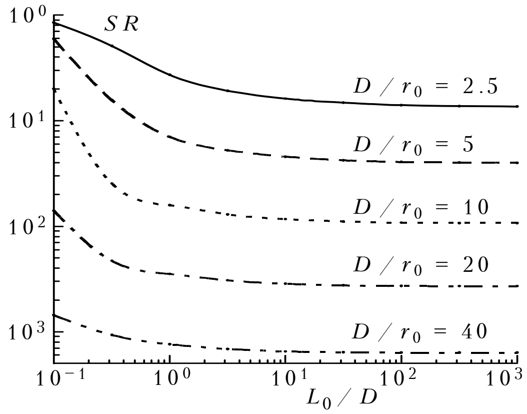

FIG. 4. Strehl parameter for a homogeneous path.

In the case of an inhomogeneous path, the calculation of SR is complicated by the fact that the value L_0 is a height function in $D_\varphi(\rho)$. Let us divide the atmosphere into the layers of dimension larger than L_0 so that L_0 can be considered constant within them. Based on the assumption that there are no correlation between turbulent fluctuations in these layers.[9,13] The complete structure function can be calculated as a sum of structure functions of every layer.

Using the obtained values L_0^* one can introduce the relative error in determination of the Strehl parameter $\varepsilon = (SR - SR^*)/SR$ where SR and SR^* are the Strehl parameters for the turbulent PSF calculated using $L_0(h)$ and L_0^*, respectively. The value ε defines the accuracy of prediction of the turbulent distortions of the image when the height profile is changed for a finite value. One can use the functions $SR(D/r_0)$ for small values of ε (Fig. 4).

The value ε is presented in Figs. 5 and 6 as a function of the diameter D and the wavelength λ. The domains of D and λ ($D \in [1,10]$ m, $\lambda \in [1,10]$ μm) correspond to real parameters of astronomical telescopes. One can note that $\varepsilon \to 0$ in the limiting cases, for very small and very large λ:

1. For very large λ, it is caused by the fact that the image is practically diffraction limited for $r_0 > D$, i.e., it does not depend on characteristics of atmospheric turbulence. This is also valid for small D.

2. For very small λ, we have $r_0 \gg L_0^*$, and SR is mainly determined by the values of $D_\varphi(\rho)$ in the coherent interval, i.e., $\rho \ll L_0$ in the given case. The behavior of $D_\varphi(\rho)$ for small values of ρ only weakly depends on L_0 because $D_\varphi(\rho)$ asymptotically tends to the power function as $\rho/L_0 \to 0$.

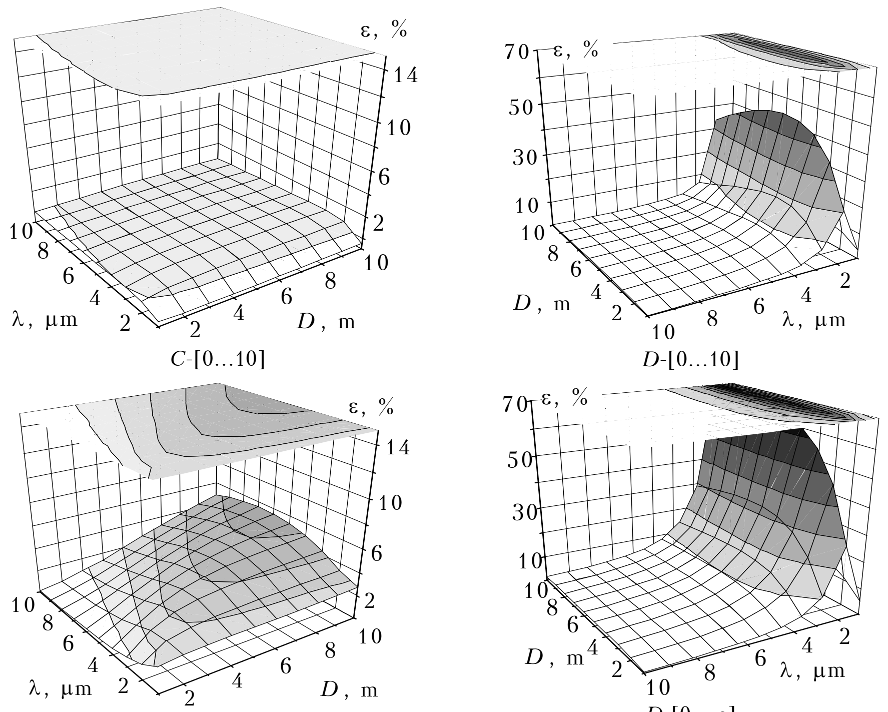

FIG. 5. Relative error of determination of the Strehl parameter under "the best" vision conditions; C, D are the types of the profile $L_0(h)$; [0...10], [0...∞] *are the methods of determination.*

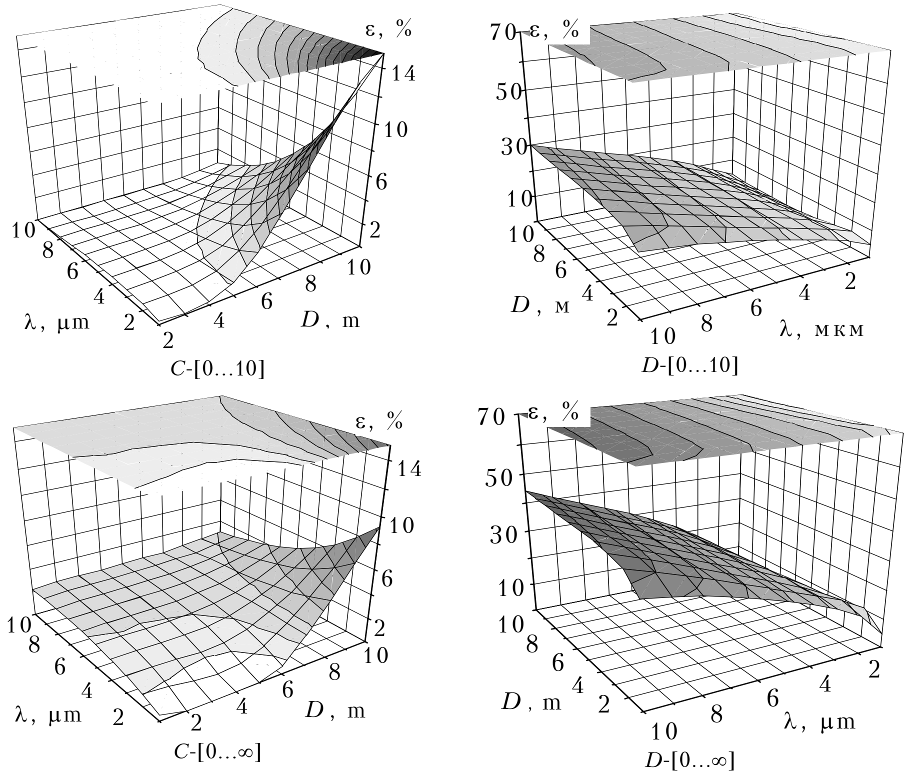

FIG. 6. Relative error of determination of the Strehl parameter under "the worst" vision conditions; C, D are the types of the profile $L_0(h)$; [0...10], [0...∞] *are the methods of determination.*

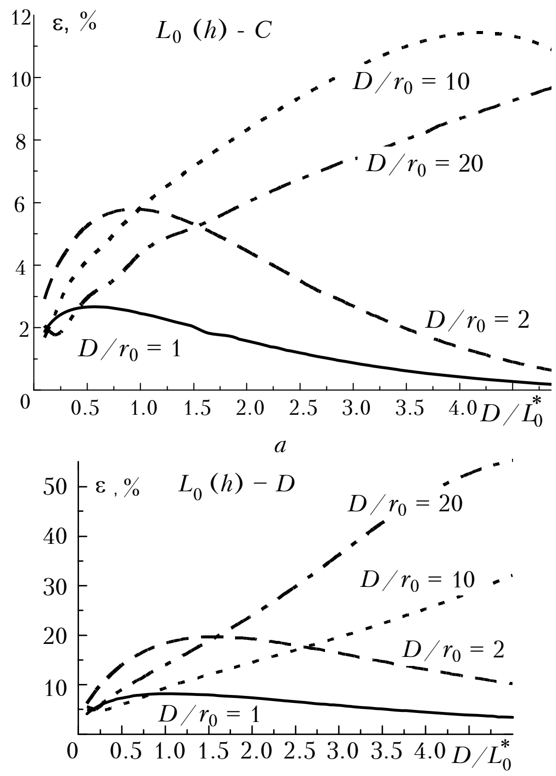

FIG. 7. Relative error ε as a function of the normalized diameter for the profile C (a) and the profile D (b).

TA” LE III.

Model $L_0(h)$	$C_n^2(h)$ – the best		
	Method		
	0...10 m	0...Arg(90%)	0...∞
(C)	0.8	4.5	6.0
(D)	50	70	80

TA” LE IV.

Model $L_0(h)$	$C_n^2(h)$ – the worst		
	Method		
	0...10 m	0...Arg(90%)	0...∞
(C)	16	11	10
(D)	30	35	45

The described behavior of ε is easily seen only under the “best” vision conditions (Fig. 5); at the same time, for the “worst” conditions (Fig. 6), these mechanisms manifest themselves only in the domain of small λ. A possible explanation is that the considered range of values of D and λ is not sufficient for this model. The characteristic behavior of ε can be seen in Fig. 7: the growth to a maximum value followed by a drop; this is especially true for small values of $D/r_0 \in [1,10]$. For $D/r_0 > 10$, the maximum value of ε can be seen when $D/L_0 > 5$. The maximum values of ε(%) are presented in Tables III and IV for two models of $L_0(h)$.

5. CONCLUSION

In the final analysis one can arrive at the following conclusions.

1. One can introduce the effective outer scale of turbulence as an integral parameter describing the character of atmospheric turbulence along the whole propagation path.

2. Introduction of the effective outer scale can considerably simplify mathematical calculations connected with the account for the influence of atmospheric turbulence on the phase of optical wave propagated along vertical atmospheric paths.

3. The description accuracy studied demonstrate that the error caused by the change of the height profile of the outer scale for a constant value, i.e., the effective outer scale, considerably varies depending both on the model of a parameter profile and the method of determination.

4. The error in determination of the Strehl parameter does not exceed 16% in the situation when the effective outer scale is larger than the diameter of a telescope.

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