ON THE DEPENDENCE OF OPTICAL REFRACTION ON ATMOSPHERIC PARAMETERS

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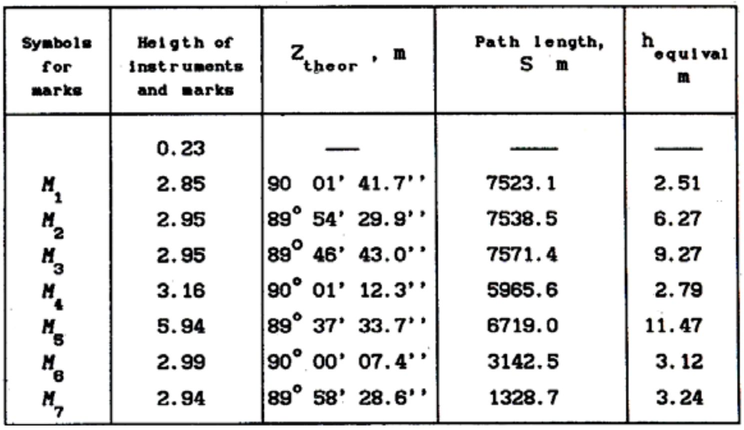
Basis meteorological atmospheric parameters such as air temperature, air pressure, cloudiness, wind-velocity, wind-direction, and vertical and horizontal temperature gradients exert direct and indirect influence on the magnitude and character of variation of the optical refraction value in the ground atmospheric layer.

It is difficult to determine the optical refraction values using meteorological measurements due to the complicated influence of the parameters mentioned above, their interaction, and the inevitable errors which result from extrapolation of measurements of meteorological quantities at one or several points along the whole path[1,2].

To study possibilities of determination of the correlations between the values of the optical refraction and the meteorological quantities, together with our colleagues at the L'vov Polytechnical Institute, we carried out an experimental investigation in the littoral area of Nikolaevsky oblast. Figure 1 and Table 1 show, respectively, a diagram of the site where the measurements were carried out and the parameters of the paths on which zenith distances were observed. Measurement paths were chosen in such a way that it was possible to determine the character of the optical refraction variations over various underlying surfaces-both land and sea. As is clear from a comparison of twenty-four hours of data of refraction angle variations for the two paths, the range of refraction angle variations for the two paths, the range of the optical refraction value oscillations over land is wider than over the sea (Fig. 2). This situation occurs mainly when the weather is clear, at which time the refraction angle over the sea varies from –0.23′ to 0.79′ at the same time over the land it changes from –0.77′ to 0.60′.

TABLE 1.



Symbols for marks	Heigth of instruments and marks	Z_{theor}, m	Path length, S m	$h_{equival}$ m
	0.23	—	—	—
M_1	2.85	90 01' 41.7''	7523.1	2.51
M_2	2.95	89° 54' 29.9''	7538.5	6.27
M_3	2.95	89° 46' 43.0''	7571.4	9.27
M_4	3.16	90° 01' 12.3''	5965.6	2.79
M_5	5.94	89° 37' 33.7''	6719.0	11.47
M_6	2.99	90° 00' 07.4''	3142.5	3.12
M_7	2.94	89° 58' 28.6''	1328.7	3.24

The refraction angle measurements were obtained using an OT-2 optical theodolite with 1.2″ + 1.7″ rms error. Meteorological observations were carried out at several points simultaneously with the refraction angle measurements. A 10-meter mast with air temperature, wind-velocity, and wind-detection, gauges (at the standard levels 0.5 m, 2 m, and 4 m) was emplaced near the main observation point. The second mast was set sea at a distance of 50–60 m from the shore and the air temperature was measured at heights of the 0.5 and 2 m above sea level (Fig. 1).

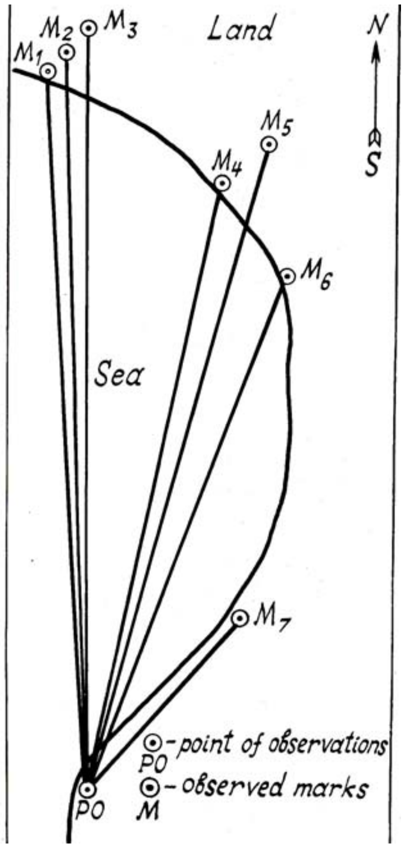


FIG. 1.

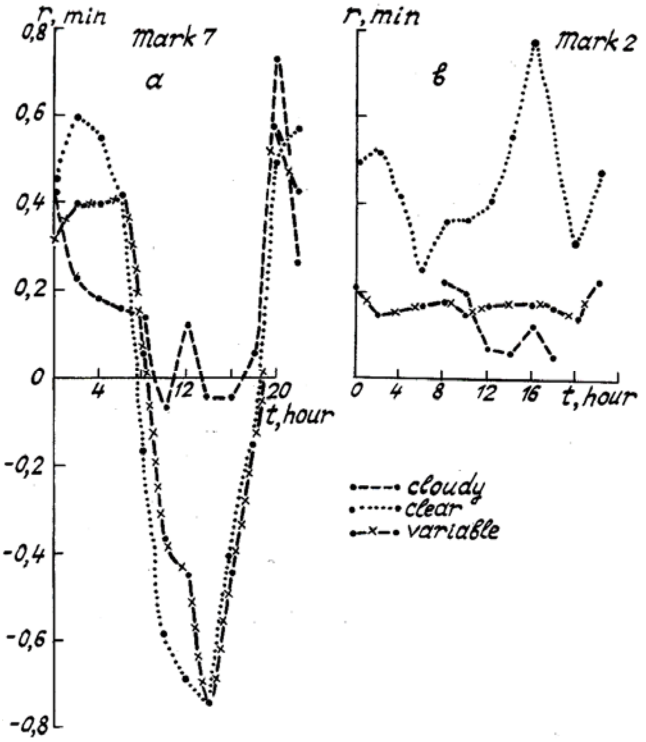


FIG.2.

The dependence of the optical refraction vertical angle on the meteorological quantities is expressed by the formula (see, e.g., Ref. 2):

$$r = 8.13\ SPT^{-2}(0,0342 + \gamma) \qquad (1)$$

where P is the air pressure (mb), T is the air temperature (°K), γ is the mean vertical air temperature gradient of along the path (deg/m).

Formula (1) lies at thed basis of almost all methods of refraction angle calculations based on the results of meteorological measurements. Extrapolation of the meteorological measurements carried out at one or several points along the whole path is the main source of error of these methods.

Therefore the determination of the correlation between the optical refraction angle values and the meteorological quantities by means of the multiply linear regression equations[3] is of great interest. For this purpose the pairwise of correlation coefficient between the path directions $K_{m_1 m_{i-1}}$ (Fig. 1) were determined from the measured refraction angles for all seven directions, and the thusly obtained correlation coefficients were estimated, i.e., the standard deviations σ_k were calculated. The results are given in Table 2, from which it can be seen that pairwise correlation coefficients very over the range 0.138 + 0.916. The values of these coefficients for directions with identical equivalent heights and for directions with unequal equivalent heights are quite high and comparatively low, respectively. For instance, the equivalent heights for the first and the fourth directions are equal to 2.51 m and 2.79 m, respectively, when the distance difference is approximately 2 km. Nevertheless, the correlation coefficient is equal to 0.916 at the same time that it is equal to 0.467 for the first and the fifth directions having an almost 9-m equivalent height difference although the distance difference is small. Such a pattern can be observed for all directions, except the sixth and the seventh ones. The sixth direction passes very close to the sea-land border and the seventh one passes over the land. The correlation connection between the seventh and all other directions is low independently of equivalent heights and pathslengths, since meteorological processes which determine the values and the nature of the refraction angle variations depend on the kind and the type of underlying surface.

TABLE 2.

Path direction		M_1	M_2	M_3	M_4	M_5	M_6	M_7
M_1	k_{M_1M}	—	0.869	0.683	0.916	0.467	0.642	0.143
	σ_k		0.025	0.037	0.016	0.076	0.055	0.089
	n		98	110	102	107	117	121
M_2	$k_{M_2M_1}$	—	—	0.850	0.873	0.563	0.806	0.146
	σ_k			0.015	0.019	0.052	0.024	0.069
	n			203	163	171	219	202
M_3	$k_{M_3M_1}$	—	—	—	0.781	0.756	0.779	0.231
	σ_k				0.029	0.027	0.028	0.065
	n				172	175	192	211
M_4	$k_{M_4M_1}$	—	—	—	—	0.321	0.657	0.173
	σ_k					0.060	0.055	0.061
	n					221	117	256
M_5	$k_{M_5M_1}$	—	—	—	—	—	0.251	0.138
	σ_k						0.068	0.070
	n						117	211
M_6	$k_{M_6M_7}$	—	—	—	—	—	—	0.208
	σ_k							0.067
	n							270

After analyzing the data of all of the optical refraction angle observations and the meteorological measurement data, we chose to look at the following four directions: the second, third, fourth and seventh because the most complete sets of observations had been obtained for these directions. Using the multiple regression equations, we calculated the pairwise correlation coefficient k between the measured refraction angle values and the basic meteorological quantities r = f(T, P, γ), the total correlation coefficient R, and the variances of these coefficients.

For the case under consideration the multiple regression equation is as follows:

$$r' = \frac{\sigma_r}{\sigma_t}\frac{D_{r,T}}{D_{r,r}}T' + \frac{\sigma_r}{\sigma_\gamma}\frac{D_{r,\gamma}}{D_{r,r}}\gamma' + \frac{\sigma_r}{\sigma_p}\frac{D_{rP}}{D_{r,r}}P', \quad (2)$$

where $r' = r_1 - r$,

$T' = T_1 - T$,

$P' = P_1 - P$,

$\gamma' = \gamma_1 - \gamma$,

and σr, σT, σP, and $\sigma\gamma$ are the standard deviations of the refraction angle, temperature, pressure, and vertical air-temperature gradient, respectively; $Dr\gamma$, DrP, DrT are the minors of determinant Drr; T, P, γ are the meteorological quantities averaged over the observation time.

The pairwise correlation coefficients between the variables under consideration are the elements of determinant D_{rr} and its minors (Table 3.)

TABLE 3.

	Path direction			
Correlation coeffisients	M_2	M_3	M_4	M_7
R	0	0	0	0.297
k_{rT}	0.242	0.060	0.412	-0.181
σ_k	0.068	0.072	0.046	0.039
n	190	203	329	613
$k_{r\Delta T}$	0.622	0.496	0.643	0.045
σ_k	0.059	0.073	0.056	0.096
n	109	109	109	109
$k_{r\gamma}$	0.144	0.058	0.069	-0.282
σ_k	0.071	0.070	0.055	0.037
n	190	203	329	613
$k_{r\gamma_M}$	0.509	0.557	0.410	0.012
σ_k	0.079	0.073	0.089	0.106
n	88	88	88	88
k_{rP}	0.502	0.333	0.325	0.026
σ_k	0.054	0.063	0.049	0.040
n	190	203	329	613

The total correlation coefficient between the dependent variables is given by[3]:

$$R = \sqrt{1-\frac{D}{D_{rr}}} \tag{3}$$

If the pairwise correlation coefficients, which characterize the degree of linear connectedness between two variables are varied from –1 to 1, then the total coefficient varies from 0 to 1.

From our calculations we found that the total correlation coefficient between the refraction angles and the meteorological quantities for the second, third and fourth directions R equal to zero.

Therefore, the pairwise correlation coefficient between the refraction angle values and the air temperature krT, pressure krP, as well as vertical air-temperature gradient are so low that there is practically no sense in composing the linear regression equation (2). For the seventh direction the total correlation coefficient is equal to 0.297. Therefore, for this path the meteorological quantities measured at the observation point can be used to determine the refraction angle.

From our calculations of the correlation dependence of the optical refraction values on the vertical ail—temperature gradient measured over the sea (Fig. 1) and the difference between the air and water temperatures, we can obtain a closer connection between the second, third, and fourth directions than for the seventh one. In conclusion we note that in determining the refraction angles along long paths from measurements of the meteorological quantities it is necessary to take into account the following circumstances. First, the points for meteorological observation should be chosen with maximum care. Second, the mean-integral value of the vertical air-temperature gradient, taking into account the influence of structure and type of surface (e.g., by the method proposed in Refs. 2 and 4) for paths passing over inhomogeneous underlying surfaces must be determined.

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