Probability density of irradiance scintillations in turbulent atmosphere

A.L. Afanas'ev, V.A. Banakh, and A.P. Rostov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk

Received August 29, 2007

Experimental data on the variance and probability density function (PDF) of strong optical irradiance scintillations in turbulent atmosphere are analyzed. It is shown, that the log-normal PDF model describes the law of strong irradiance scintillations distributions better than *K*-PDF model in a wide variation range of the parameter β_0^2, characterizing turbulent propagation conditions.

Introduction

Turbulent fluctuations of the air refractive index result in random distortions of light waves propagating in the atmosphere. Degradation of laser beam spatial coherence and irradiance scintillations appear. This requires the statistical description of regularities in the optical radiation propagation in the turbulent atmosphere.

Only several first statistical moments of intensity are to be known in many cases for theoretical modeling of the light propagation in the random medium and for design of atmospheric optical systems. However, this is insufficient for evaluation of reliability and noise immunity of optical communication lines, noise level calculations in optical theta and precision ranging systems. In these cases, it is necessary to know the distribution law or PDF of irradiance scintillations in the turbulent atmosphere, which is the fullest single-point statistical characteristic of a random process.

Many theoretical and experimental works are devoted to probability distribution laws of turbulent irradiance scintillations. It has been ascertained and now is indisputable that the log-normal law is valid within the domain of weak irradiance scintillations, when $\beta_0^2 < 1$.[1] The parameter

$$\beta_0^2 = 1.23 C_n^2 k^{7/6} x^{11/6},$$

where C_n^2 is the structure characteristic of the air refractive index; $k = 2\pi/\lambda$ characterizes the turbulent atmosphere conditions for propagation of optical radiation with the wavelength λ along a path of x length.

The probability distribution law of turbulent irradiance scintillations within the focusing domain $\beta_0^2 \sim 1$ and the domain of strong turbulent irradiance scintillations $\beta_0^2 > 1$ has been ascertained with a lower definiteness. The relative variance of irradiance scintillations $\sigma_I^2 \to 1$ at $\beta_0^2 \to \infty$, therefore, the single-sided exponential distribution was discussed in a number of works (see, e.g., Refs. 2 and 3) as the probability distribution law of turbulent irradiance scintillations. This model is criticized in Refs. 3–5.

The *I*–*K*-distribution model,[6] changing to *K* distribution[7,8] at strong optical turbulence on the path, is wider used. The applicability of *I*–*K*, *K*, and other distribution types to description of the probability distribution of irradiance scintillations in turbulent atmosphere were theoretically and experimentally studied in Refs. 9–13.

Extensive experimental researches of this process have been carried out during two last decades of the last century at the Institute of Atmospheric Optics SB RAS.[14–28] However, despite the large number of works, an unambiguous conclusion has not been drawn about the applicability of one or another model for description of PDF of strong irradiance scintillations on atmospheric paths. Though, due to their simplicity, *K* and single-sided exponential distributions are used when estimating parameters of atmospheric-optical communication lines (see, e.g., Refs. 29 and 30), it has been shown[31,32] on the base of numerical modeling of optical radiation propagation in turbulent atmosphere, that the *K* distribution describes strong irradiance scintillations much worse than, e.g., Beckmann distribution[33] or the logarithmically modulated exponential one.[9]

Note, that Refs. 9–11 and 32 include sufficiently full foreign bibliography and reflect the history of the related investigations.

An ambiguity of conclusions of theoretically constructed probability distribution laws and the need in practical use of atmospheric optical systems determine the present urgency of experimental researches.

The digital recording equipment, used by us in 1983–1998 [Refs. 14–28] did not give an exact pattern of the distribution laws of strong irradiance scintillations in both signal fading and burst regions because of the limitation of the equipment dynamic range.[34–36] The improved dynamic parameters of the equipment allowed us to obtain qualitatively differing data, which introduced some uncertainty in their interpretation.

0235-6880/08/02 102-05 $02.00

Thus, irradiance scintillations of a spherical wave, reflected from a two-dimensional array of prismatic angular reflectors were described in Ref. 20. While studying, the conclusion has been drawn that the PDF of weak irradiance within the signal fading region essentially differs from the log-normal model and better agrees with the universal one, while the experimental histograms for strong scintillations are satisfactorily approximated by the universal and *K* distributions. It has been also shown in Ref. 21 that experimental probability distributions of strong irradiance scintillations of a plane wave deviate from the log-normal one and are better described by the *K* distribution. When studying plane and spherical waves, reflected from the angular reflector array,[28] the conclusion on *K* distribution preference was also drawn.

In Refs. 14–28, probability laws of irradiance scintillations were concluded from the analysis of intensity histograms measured with equipment having a sufficient dynamic range, which is of great importance. In 1999, we designed an analog recorder with a dynamic range of 96 dB. Using this recorder, we repeated a number of past experiments and obtained results, presented in this work.

Description of experiment and used equipment

The measurements were carried out in June–July, 1999, in the afternoon on a horizontal path above a plane underlying surface. The optical schematic of the experiment is shown in Fig. 1.

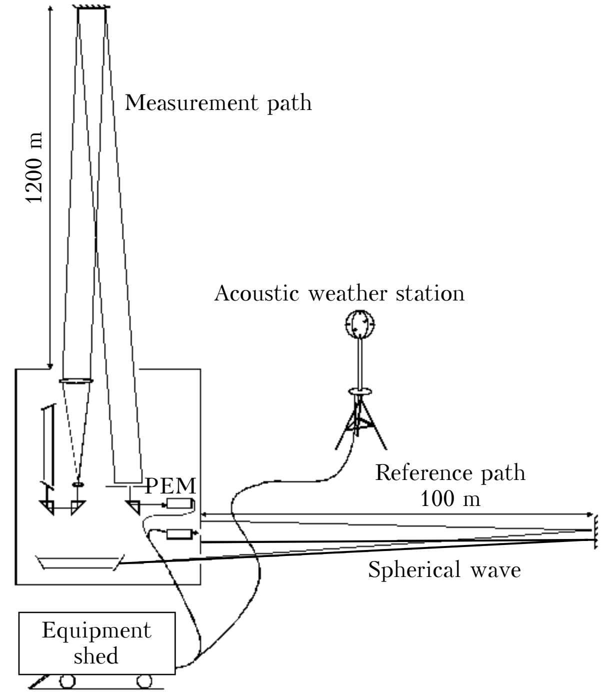

Fig. 1. Optical schematic of the experiment on studying the distribution laws of laser irradiance scintillations.

The He–Ne laser irradiance (0.63 μm) was directed through a 500-mm collimator to the reflector, 1200 m distanced from the source. As the reflector, a plane mirror of 500 mm in diameter was used. The type of laser wave varied from a focused beam to a plane one by means of the collimator lens. The wave was spherical in the absence of the collimator lens. The reflected radiation was received with a PEM-79 photodetector with the entrance aperture of 0.3 mm in diameter, which was 1500 mm distanced from the collimator optical axis.

An electric signal from the PEM arrived at one of the channels of a laptop digital recording complex, specially designed for the experiment. The dynamic range of the complex was 96 dB. The block-diagram of the complex is shown in Fig. 2.

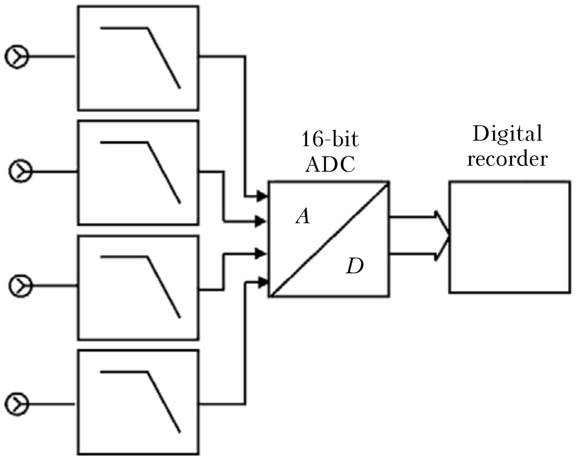

Fig. 2. Block-diagram of the complex for analog signal recording.

Before the analog-to-digital conversion, signals passed through a tenth-order Butterworth hardware low-pass filter (LPF) (with maximum flat response). The cutoff frequency was 1 kHz with a quenching of 68 dB per octave. Then signals were converted into digital form with a 4-channel 16-bit analog-to-digital converter (ADC) with an accuracy and linearity equal to the half of the least significant bit. After ADC output, a signal in the form of digital data stream was written in a storage of a software digital recorder. As is well known, the probability of writing and data storage errors in a PC is negligible.

The control program language is Assembler, which allows minimization of PC processor operation delays in order to maximize the speed and epoch accuracy. While operating, the program disables all the hardware PC interrupts and controls the hardware, thus turning the PC into a simple digital data recorder.

On completion the work, the obtained data array, a little exceeding 9 MB in size, was written from the recorder ram to the PC HDD by DOS means. The recording error probability was reduced to zero in contrast with previous experiments, where the error was 10^{-6} [Ref. 34].

In the experiment, a signal was recorded during 5 min with a sampling rate of 5 kHz. When recording at the switched on laser, the average signal level varied, because in the PEM field of vision the object illuminance depended on variations of the solar light incidence angle and the atmospheric transmittance. This effect was not pronounced in previous experiments, because the sensitivity of the used equipment was 16 times lower; hence, the laser beam was intercepted before and after each realization and flare backlight was measured. While processing, the linear trend of recording-time average signal level was subtracted from the recorded realization. In addition, PEM noises were recorded in the absence of laser signal to obtain more reliable PDF values within the fading region. In the following processing, this signal was excluded with the help of convolution[37] of irradiance scintillation histogram with the backlight PEM noise one:

$$I = I_{\text{s}} - I_{\text{n}},$$

$$f(I) = \int_{-\infty}^{\infty} f_{\text{s}}(I_{\text{s}}) f_{\text{n}}(I_{\text{s}} - I)\, \mathrm{d}I_{\text{s}},$$

where I_{s} is the recorded signal and I_{n} is the PEM noise.

To control turbulent conditions of the atmosphere, the structure constant of refraction function C_n^2 was used. Data for its calculation were obtained from measurements of irradiance scintillations of a spherical wave σ_I^2 on an individual V-shape path of 200 m in total length. Saturation of the irradiance scintillations does not occur at such path at turbulence levels, realizable in the atmosphere. In this case, the dependence of C_n^2 on σ_I^2 is well described by the equation, obtained in the first approximation of the method of smooth perturbations[1]:

$$C_n^2 = \frac{1}{0.344} \sigma_I^2 k^{-7/6} x^{-11/6},$$

where

$$\sigma_I^2 = \frac{<I^2>}{<I>^2}.$$

A He–Ne laser free of forming optics was used as an emitter, and a PEM-79 with an entrance aperture of 0.5 mm in diameter – as a radiation receiver. Signals from PEM arrived through the amplifier to the second ADC channel.

This technique has some advantages over the gradient and local meters due to spatial averaging and efficiency at small gradients of temperature and wind speed, when local meters give larger errors. Additional control of turbulence stability carried out with an ultrasonic anemometer-thermometer, 50-m distanced from the instrumentation shed.

Experimental data analysis

Totally, sixty seven measurement runs with different wave types were carried out. For each realization, histograms of measured intensities were analyzed, the normalized irradiance scintillation moments $M^n = m_n / m_1^n = <I^n> / <I>^n$ were calculated, and parameters β_0 and C_n^2 were assessed. Weather conditions were also monitored. The maximum of β_0 was 9, while its average value did not exceed 0.5 on the reference path, used for measuring C_n^2.

Characteristic histograms for spherical and plane waves, as well as focused beam are shown in Fig. 3.

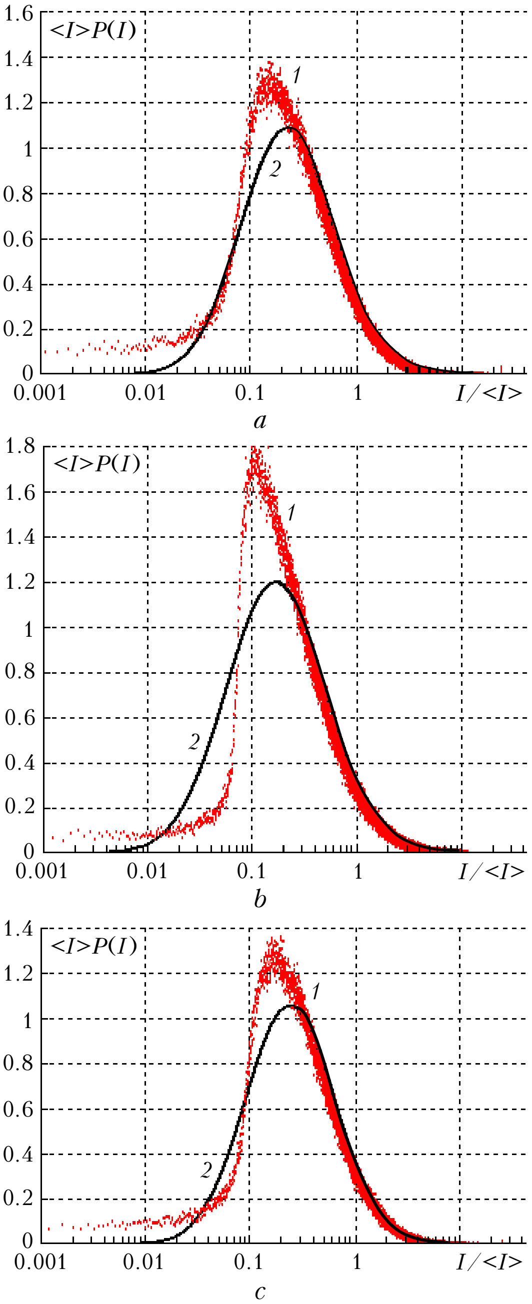

Fig. 3. Intensity histograms for spherical (*a*) and plane (*b*) waves and focused beam (*b*): experiment (*1*) and log-normal distribution (*2*).

Theoretical analysis of the PDF of strong irradiance scintillations is mainly based on estimation of statistical moments of intensity. However, in actual atmospheric experiments, estimations of higher-order moments can have significant errors[26] due to limitation of the recorder's dynamic range. An equation for estimation of the nth truncated moment bias (corresponding to real measurements) relative to the model one for log-normal distribution was derived in Ref. 15.

The dependence of the higher normalized experimental moments M_n ($n = 3, 4, 5$) on the second one M_2 is shown in Fig. 4. Signs *1–3* designate the experimental moments. Curves *4–6* correspond to log-normal distribution moments calculated with accounting for the truncated moment estimation bias.[15]

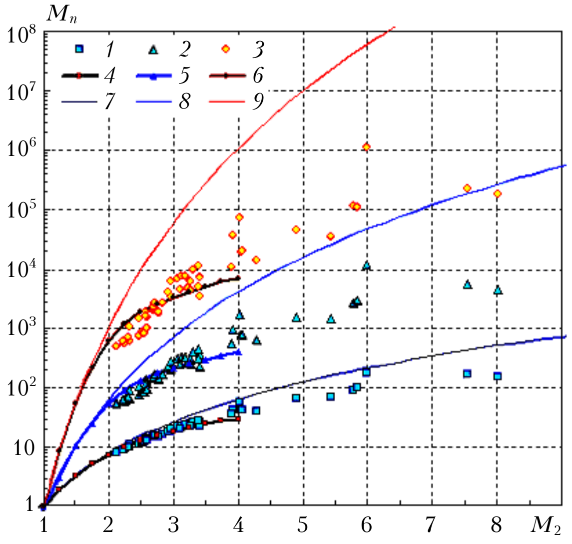

Fig. 4.

Curves *7–9* correspond to the theoretical dependence for the log-normal distribution

$$m_n = \langle I^n \rangle = \exp\left(\xi n + \sigma^2 n^2 / 2\right),$$

where $\xi = \ln(\langle I\rangle / \sqrt{M_2})$ is the mean and $\sigma^2 = \ln(M_2)$ is the normal distribution variance.

The data analysis by histograms and moment methods with consideration of the whole range of values taken by the random process during measurements, unambiguously shows that the log-normal model for the probability density describes the distribution of irradiance scintillations within the signal fading region better than K distribution. According to numerical calculations, the PDF values for K distribution at minimal experimentally observed normalized intensities $I/\langle I\rangle = 10^{-3}$ are a little lower than in maximum, while the experimental data and log-normal distribution give virtually zero PDF values already at $I/\langle I\rangle = 10^{-2}$.

The questions of which model of PDF distribution law approximates experimental distributions more closely and whether logarithmically modulated normal exponential[9] and Beckmann[33] distributions can be really acceptable for the PDF of strong irradiance scintillations in turbulent atmosphere (as it follows from numerical experiments[32]) are the subject of further investigations.

References

1. V.I. Tatarskii, *Wave Propagation in Turbulent Atmosphere* (Nauka, Moscow, 1967), 548 pp.
2. D.A. DeWolf, J. Opt. Soc. Am. **59**, 1455–1460 (1969).
3. A.S. Gurvich, A.I. Kon, V.L. Mironov, and S.S. Khmelevtsov, *Laser Radiation in Turbulent Atmosphere* (Nauka, Moscow, 1976), 278 pp.
4. K.S. Gochelashvili and V.I. Shishov, *Scientific and Technology Results. Radiophysics. Physics for Electronics. Acoustic. Vol. I. Waves in a Randomly-Inhomogeneous Medium* (VINITI, Moscow, 1981), 144 pp.
5. V.U. Zavorotnyi, V.I. Klyatskin, and V.I. Tatarskii, Zh. Eksp. Teor. Fiz. **73**, 481–497 (1977).
6. L.C. Andrews and R.I. Phillips, J. Opt. Soc. Am. A **2**, 160–163 (1985).
7. E. Jakeman and P. Pusey, Phys. Rev. Lett. **40**, 546–550 (1978).
8. G. Parry and P.N. Pusey, J. Opt. Soc. Am. **69**, 796–798 (1979).
9. J.H. Churnside and R.J. Hill, J. Opt. Soc. Am. A **4**, No. 14, 727–733 (1987).
10. J.H. Churnside and S.F. Clifford, J. Opt. Soc. Am. A **4**, No. 10, 1923–1930 (1987).
11. J.H. Churnside and R.G. Frehlich, J. Opt. Soc. Am. A **6**, No. 11, 1760–1766 (1989).
12. R.J. Hill and R.G. Frehlich, Appl. Opt. **35**, No. 6, 986–997 (1996).
13. E.P. Milyutin and Yu.I. Yaremenko, Radiotekhn. Elektron. **25**, No. 11, 2273–2278 (1980).
14. G.Ya. Patrushev, A.I. Petrov, and V.V. Pokasov, Izv. Vyssh. Uchebn. Zaved. SSSR, Ser. Radiofiz. **26**, No. 7, 823–831 (1983).
15. G.Ya. Patrushev, T.P. Pecherkina, and A.P. Rostov, Avtometriya, No. 3, 22–27 (1985).
16. G.Ya. Patrushev and A.I. Petrov, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana **22**, No. 10, 1050–1059 (1986).
17. G.Ya. Patrushev, A.I. Petrov, and V.V. Pokasov, Opt. Spekstrosk. **61**, No. 3, 587–590 (1986).
18. A.G. Borovoy and G.Ya. Patrushev, Appl. Opt. **27**, No. 17, 3704–3714 (1988).
19. A.P. Ivanov, G.Ya. Patrushev, and A.P. Rostov, Atmos. Opt. **2**, No. 9, 774–778 (1989).
20. G.Ya. Patrushev, A.I. Petrov, and O.A. Rubtsova, Atmos. Opt. **2**, No. 3, 221–226 (1989).
21. P.A. Konyaev, V.P. Lukin, G.Ya. Patrushev, and S.Yu. Tabakaev, Atmos. Opt. **3**, No. 12, 1194–1197 (1990).
22. A.S. Gurvich, A.P. Ivanov, S.S. Kashkarov, G.Ya. Patrushev, and A.P. Rostov, Atmos. Oceanic Opt. **5**, No. 1, 29–32 (1992).
23. G.Ya. Patrushev, Atmos. Oceanic Opt. **5**, No. 3, 160–161 (1992).
24. G.Ya. Patrushev and O.A. Rubtsova, Atmos. Oceanic Opt. **5**, No. 7, 453–455 (1992).
25. G.Ya. Patrushev and O.A. Rubtsova, Atmos. Oceanic Opt. **6**, No. 11, 760–769 (1993).
26. G.Ya. Patrushev and O.A. Rubtsova, Atmos. Oceanic Opt. **6**, No. 11, 794–797 (1993).
27. G.Ya. Patrushev, A.P. Rostov, and O.A. Rubtsova, Atmos. Oceanic Opt. **8**, No. 6, 423–426 (1995).

28. A.P. Rostov and O.A. Rubtsova, Atmos. Oceanic Opt. **9**, No. 8, 683–685 (1996).

29. M. Uysal, S.M. Navidpour, and J. Li, IEEE Commun. Lett. **8**, No. 10, 635–637 (2004).

30. A.G. Zambrana, IEEE Commun. Lett. **11**, No. 5, 390–392 (2007).

31. S.M. Flatte, Ch. Bracher, and G.-Yu. Wang, J. Opt. Soc. Am. A **11**, No. 7, 2080–2092 (1994).

32. R.J. Hill and R.G. Frehlich, J. Opt. Soc. Am. A **14**, No. 7, 1530–1540 (1997).

33. P. Beckmann, *Probability in Communication Engineering* (Harcourt, Brace & World, New York, 1967).

34. A.I. Isakova, E.A. Monastyrnyi, G.Ya. Patrushev, A.I. Petrov, T.P. Pechrkina, A.P. Rostov, and V.D. Teushchekov, Avtometriya, No. 4, 15–21 (1987).

35. A.P. Rostov, Atmos. Opt. **1**, No. 3, 125–126 (1988).

36. G.F. Vakurov, L.A. Zharkova, A.I. Isakova, E.A. Monastyrnyi, G.Ya. Patrushev, and A.P. Rostov, "*Hardware-Software complex for studying optical radiation propagation in the surface air,*" Preprint No. 14, TF SB SA USSR (1984), 39 pp.

37. I.N. Bronshtein and K.A. Semendyaev, *Reference Book on Mathematics* (Nauka, Moscow, 1980), 947 pp.