METHODS OF SYSTEM ANALYSIS IN ECOLOGY OF THE URBAN ATMOSPHERE

R.N. Efremov

Russian State Institute on Hydrology and Meteorology, St. Peterburg
Received November 15, 1993

Techniques have been developed that can be used in an automated system for monitoring of the pollution of the urban atmosphere based on a radically changed approach to air pollution modelling. Primary emphasis is placed upon the information obtained from observations. It permits one to avoid the calculation of a great many starting parameters that are difficult to determine.

Methods of system analysis are used to construct the design and optimal operation algorithms for systems of air pollution monitoring.

Recently much attention has been given to the development of a system of monitoring of air pollution of the urban atmosphere. In this connection the need for the development of devices that are capable of measuring the level of pollution to a sufficient accuracy should be primarily pointed out. Another important problem is the construction of mathematical models required for the computer–controlled operation of monitoring and control system. This system must not only yield information about the air pollution, but also provide its monitoring. In this connection the system of models must solve not only direct problems on assessment of the air pollution and its prediction but also inverse problems to answer the question what to do to maximize or minimize some efficiency characteristics.

The need for the inverse problem solution imposes specific demands on the models employed according to the management science. These models must be not too cumbersome since this leads to weak sensitivity to the applied controls.

1. There are a number of theoretical models describing the scattering of impurities in the urban atmosphere. Let us model the emission of impurities into the urban atmosphere as a result of operation of a given set of continuous point sources. Since we intend to perform the model calculations on a computer, the motor transport pollution (continuous linear sources) can be modelled by a set of point sources.

Modeling is based on the following formula for the surface pollution from the jth source:

$$q_j(x, y) = \frac{A_j}{(x - x_j)^2} \exp\left\{ -\frac{B}{x - x_j} - \frac{C(y - y_j)^2}{(x - x_j)^2} \right\}, \quad (1)$$

where $q_j(x, y)$ is the surface concentration of impurities from the jth source; A_j, B, and C are the coefficients depending on meteorogical conditions; A_j is proportional to the jth source strength; and, x_j and y_j are the coordinates of the jth source. The OX axis follows the wind direction, and the OY axis is perpendicular to the OX axis.

The total pollution from n sources is calculated using the formula

$$q = \sum_{j=1}^{n} q_j = \sum_{j=1}^{n} \frac{A_j}{(x - x_j)^2} \exp\left\{ -\frac{B}{x - x_j} - \frac{C(y - y_j)^2}{(x - x_j)^2} \right\}. \quad (2)$$

According to author's technique, the coefficients A_j, B, and C are not calculated theoretically but are empirically determined, hence formulas (1) and (2) become empirical ones. The pollution densities q_k ($k = 1, 2, ..., m$) measured at the kth observation station provide a basis for calculations. The coefficients are calculated by the least square method, with B and C values being calculated for fixed period of observation by the gradient method. The coefficients A_j ($j = 1, 2, ..., n$) are calculated using the analytical formulas obtained by the author on the basis of the least square method. The coefficients A_j, B, and C as a whole are calculated by the successive approximation method. The appropriate computer program has been developed.

We can go from the coefficients A_j to the source strengths Q_j. In the particular case this permits us to implement the algorithm for the detection of a source of unauthorized emission. If q is set equal to maximum permissible concentration at the input of the above–discussed algorithm, at the output we will obtain the amount of maximum permissible emission for each source.

2. Let us transform formula (2) in the following way

$$q = \sum_{j=1}^{n} A_j f_j (x - x_j, y - y_j), \quad (3)$$

where

$$f_i(x, y) = \frac{1}{(x - x_j)^2} \exp\left\{ -\frac{B}{x - x_j} - \frac{C(y - y_j)^2}{(x - x_j)^2} \right\}. \quad (4)$$

Let us assume that the νth source exhausted an unauthorized emission, then at the kth point of observation

$$\Delta q(x_k, y_k) = f_\nu(x_k - x_\nu, y_k - y_\nu) \Delta A_\nu, \ (k = 1, 2, \dots, m). \quad (5)$$

It then follows that ΔA_ν is equal to

$$\frac{\Delta q_k}{f_\nu(x_k - x_\nu, y_k - y_\nu)}. \quad (6)$$

0235-6880/94/02 145-04 $02.00

Theoretically if the serial number of the source is chosen correctly, then ratio (6) will be independent of the number k [that is, the variance equals zero for expression (6)]. Practically this manifests itself in the minimum variance (6) for arbitrary k. The mean value of expression (6) has the form

$$\frac{1}{m}\sum_{k=1}^{m}\frac{\Delta q_k}{f_\nu(x_k-x_\nu,\, y_k-y_\nu)}\,. \qquad (7)$$

Dropping the constant factor, we obtain the following expression for the statistical variance

$$D(\nu)=\sum_{k=1}^{m}\left(\frac{\Delta q_k}{f_\nu(x_k-x_\nu,y_k-y_\nu)}-\frac{1}{m}\sum_{k=1}^{m}\frac{\Delta q_k}{f_\nu(x_k-x_\nu,y_k-y_\nu)}\right)^2. \qquad (8)$$

This function will be minimum when the number ν corresponds to the number of source of unauthorized emission. The computer program has been developed which implements this algorithm.

3. Now we consider the probabilistic approach to the problem of detection of the source of unauthorized emission. If the number of atmospheric air pollution observations is sufficiently large, one can determine in a statistical way $P(H_j)$, that is, the probability of the emission from the source with serial number j, and the distribution densities $f_{\Delta A_j}(\nu)$ (j = 1, 2, ..., n).

Since according to relation (5) Δq_k depends linearly on ΔA_j, we may derive the distribution law for Δq_k from that for ΔA_j

$$f_{\Delta q_k}(\Delta q_k/H_j)=f_{\Delta A_j}\left(\frac{\Delta q_k}{\varphi_j(x_k-x_j,\, y_k-y_j)}\right)\frac{1}{\varphi_j(x_k,\, y_k)}. \qquad (9)$$

Furthermore we obtain Δq_1, Δq_2, ... Δq_k, after which the generalized hypothesis formula can be applied

$$P(H_j/\Delta q_1,\Delta q_2,\ldots\Delta q_k)=\frac{P(H_j)f(\Delta q_1/H_j)\ldots f(\Delta q_k/H_j)}{\sum_{j=1}^{n}P(H_j)f(\Delta q_1/H_j)\ldots f(\Delta q_k/H_j)}, \qquad (10)$$

(j = 1, 2, ... , n) .

Comparing this conditional probabilities, we can obtain the solution of the problem on detection of the source of unauthorized emission.

4. Now there are a sufficiently large number of papers dealing with the optimal location of the pollution observation stations (see, for example, Refs. 1 and 2). Many authors extend the consideration for the field variation previously applied in meteorology to the fields of meteorological elements. But it seems more appropriate to use another approach based on the treatment of the pollution field as a result of operation of the fixed number of continuous sources.

In this connection let us characterize every point of a town by the average number of sources to which the observation station, located at this point, is sensitive. Upon integrating formula (1) over wind speed and direction and taking into account the recurrence of these parameters throughout the year, we obtain the mean concentration from a given source located at the point with (x_{ij}, y_{ij}) coordinates. Modeling of every source enables us to find the number of sources a_{ij} which contribute to the pollution at this point.

We consider the model of a town as the totality of $m\times n$ elements. Every element is characterized by the (x_{ij}, y_{ij}) point. Let X_{ij} be equal to 1, if the observation point is to be placed in this element, or 0, if there is no such a need. The calculated number a_{ij} is affixed to every point. In calculations only that sources are taken into account whose contribution to the pollution level at the examined point exceed a fixed threshold value.

Let us write the average number of sources detected at one observation station as

$$\frac{1}{M}\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}X_{ij}\,, \qquad (11)$$

where M is the total number of the observation stations. Let us finally formulate the following problem which enables us to place the observation stations. It is necessary to find a maximum

$$Q=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}X_{ij} \qquad (12)$$

with restrictions

$$\sum_{i=1}^{m}\sum_{j=1}^{n}X_{ij}=M\,,\quad \sum_{i=1}^{m}X_{ij}\ge 1,\quad \sum_{j=1}^{n}X_{ij}\ge 1\,. \qquad (13)$$

The first restriction means that the number of the observation stations equals M, the second and third restrictions provide the distribution of the observation stations over the territory of a town.

The above–formulated problem can be solved by the simplex method of linear programming.

TABLE I. Matrix of detectable sources.

0.4	0.5	0.2	0.2	0.4	0.3
0.4	0.4	0.3	0.3	0.3	0.3
[1.2]	0.4	0.5	0.6	0.4	0.3
0.4	0.4	0.2	0.4	0.5	**[1.2]**
0.9	0.3	0.1	0.2	0.5	0.3
0.9	**[1.2]**	0.9	0.3	0.3	0.3
0.9	1.2	**[1.2]**	0.3	0.3	0.3

In the particular case in which we may neglect the second and third restrictions the problem reduces to the following simple procedure. Let us calculate the field a_{ij} and find the maximum a_{ij}. The first observation station corresponds to this maximum. The second observation station corresponds to the maximum a_{ij} on condition that a_{ij} for the first observation station is excluded from consideration, and so on. The computer program has been developed by the author to solve the above–discussed problem. The results of computations by the developed technique are shown in Table I. It was suggested that four observation stations should be located in the town. The framed numbers correspond to the chosen stations.

5. The problem of monitoring of the urban air pollution is one of the most interesting problems. We

consider first of all the problem of monitoring of the pollution from n continuous point sources. Let u_j be the relative decrease of the emission strength from the jth source, $\varphi_j(u_j)$ be the expense of the emission strength decrease, and ν_j be the maximum decrease of the strength emission from the jth source, with

$$0 \le u_j \le \nu_j \le 1 \ . \tag{14}$$

If monitoring is carried out, then the impurity concentration at the (x, y) point has the form

$$q(x, y) = \sum_{j=1}^{n} A_j\,(1 - u_j)\, f_j\,(x - x_j\,,\, y - y_j)\ . \tag{15}$$

We may formulate the following problem of monitoring: find such a vector $\mathbf{u} = (u_1, u_2, ..., u_n)$ which minimizes the expense of monitoring

$$\Phi(\mathbf{u}) = \sum_{j=1}^{n} \varphi_j(u_j) \tag{16}$$

with the restrictions

$$\sum_{j=1}^{n} A_j\,(1 - u_j)\, f_j\,(x_k - x_j,\, y_k - y_j) \le \text{MPC}\ (k = 1, 2, ..., m),$$

$$0 \le u_j \le \nu_j, \tag{17}$$

where m is the number of the observation stations.

The problem of monitoring has been formulated in the form of the linear programming problem and can be solved by the simplex method.

Now we consider another problem of monitoring. Let the resources assigned for monitoring are limited, that is, $c \le c_0$. What is wanted is a control vector $\mathbf{u}$ which minimizes the mean level of pollution

$$P(\mathbf{u}) = \frac{1}{m} \sum_{k=1}^{m} \sum_{j=1}^{n} A_j\,(1 - u_j)\, f_j(x_k - x_j,\, y_k - y_j) \tag{18}$$

with restriction $\sum_{j=1}^{n} \varphi_j(u_j) \le c_0$. This problem can also be solved by the simplex method. The computer program has been developed by the author for the first of the above–discussed problems.

6. A special feature of the motor transport air pollution, connected with the fact that the decrease of the strength of linear sources $Q_j(0)$ on one main road leads to the increase of the amount of emissions on the others, must be taken into account in air pollution monitoring. The values of $Q_j(0)$ can be determined by the adaptation method (see item 1) and on the basis of the experimental data about the flow of traffic converting it to $Q_j(0)$ by the known methods.

Now we consider the method of statistical simulation of the flow of traffic. We simulate the initial and final points of the route (the simplest assumption is the uniform distribution law for corresponding random vectors). The route between these two points is specified by the dynamic programming method. Repeat these procedures n times. Summing over the route lengths and dividing the sum by the mean velocity of motion, we obtain the mean duration of one trip T.

Then we calculate the number of routes passing through the given point for all chosen points. Dividing this value by T, we obtain the ancillary flow of traffic $\mu(x, y)$ at the (x, y) point. Let N be the number of cars in a town, with ν indicating what part of cars is in motion. The resultant flow of traffic, 1/time unit, is calculated from the formula

$$M(x, y) = \mu(x, y)\,(N\,\nu/n)\ . \tag{19}$$

Furthermore, the flow of traffic converts to the strength of emissions.

Let us assume that

$$z_j = \begin{cases} 0\ , \text{ when the traffic along the } j\text{th street is blocked} \\ 1\ , \text{ otherwize}\ , \end{cases}$$

and denote the length of the jth street by l_j. Let the goal function

$$G = \sum_{j=1}^{m} z_j\, l_j \tag{20}$$

be the total length of streets opened for traffic, and m be the total number of streets. Then the problem of monitoring can be formulated in the following way: to find $z_j (j = 1, 2, ..., m)$ which maximizes G on condition that the pollution level at the observation stations does not exceed the maximum permissible concentration (MPC).

7. The urban air pollution can also be monitored in time. We adopt the following simple assumptions. Each jth source must operate the fixed time Δa_j. The beginning of operation of the jth source can be chosen in the time interval $[a_{j_1}, a_{j_2}]$, but in such a way that Δa_j falls within the interval $[0, T]$.

One possible variant of operation of the controllable sources is shown in Table II (Gant's plot). Double dash denotes the time when the source is operating.

TABLE II. One possible variant of operation of controllable sources.

Serial number of the source	Hours							
	1	2	3	4	5	6	7	8
1	=	=						
2		=	=	=	=			
3							=	=
4				=	=	=	=	

The definite function $q(x, y, t)$ of three variables, that is, the urban air pollution level, corresponds to each variant.

The expression

$$G_1 = \max_{x,\, y,\, t}\, q(x, y, t) \tag{21}$$

is taken as the goal function .

The problem of monitoring consists in determination of the variant of the source operation that satisfies the above–formulated restrictions and minimizes the quantity G_1. The operation variant which minimizes the mean level of the urban air pollution

$$G_2 = q\left(\overline{x, y, t}\right)$$

is also determined in calculations, where averaging is performed over time and coordinates x and y.

The above–formulated problems of monitoring are the scheduling problems most difficult for the solution.[3]

Let us solve these problems by the statistical simulation method considering the start of operation of the jth source to be uniformly distributed in the interval $[0, T - \Delta a_j]$. This permits us to obtain one realization of the schedule and to find the corresponding value of the goal function. Quasioptimum schedule will correspond to the minimum goal function. The program implementation of the above–discussed algorithm has been developed by the author.

8. The technique for choosing of the location of the pollution observation stations has been developed in item 4. Now we consider the problem of the optimum route choice for the mobile station of pollution control. Assume that this station must carry out the observations in n points in addition to the stationary stations. Let us specify in item 4 the concrete meteorological conditions corresponding to the pollution observation. We exclude for a while from consideration the regions of the town, which have stationary observation stations. Using the procedure of item 4, we obtain the coordinates of points at which additional observations are needed. Then it is necessary to solve the problem of commercial traveller for additional stations and starting point of the route: how to travel all over the indicated points with minimum travelling expense? For this problem a computer program has been developed using the branch and bound algorithm[3] providing the cheapest travelling all over the additional observation points.

Example. On the basis of the technique described in item 4, the coordinates of the observation points have been calculated for mobile station. These data provide the basis for calculation of the matrix of the mobile station travelling expense from the ith (row number) to the jth (column number) observation point

$$\begin{pmatrix} x & 1 & 4 & 12 \\ 2 & x & 5 & 8 \\ 3 & 6 & x & 10 \\ 11 & 7 & 9 & x \end{pmatrix}$$

(station starts from the first point). Thus it is seen from the results of our calculation that the cheapest route is the route through 1–2–4–3–1 points and the minimum expense is equal to 21 units.

REFERENCES

1. V.V. Panenko, V.F. Raputa, and A.V. Bykov, *Application of the Methods for Experimental Design in Problems on Assessment of the State of Hydrological and Meteorological Fields* (Nauka, Novosibirsk, 1986), 19 pp.

2. A.V. Primak, in: *Proceedings of the Second All–Union Conference,* Leningrad, 1986, pp. 41-51.

3. H. Takha, *Investigation of Operations* [Russian translation] (Moscow, Mir, 1985), Vol. 1, 479 pp.;Vol. 2, 495 pp.