A COMPARISON OF METHODS OF CALCULATING THE RADIATION FLUXES IN CUMULUS CLOUDS

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The accuracy of an approximate methods used to calculate the mean value, variance, and correlation function of the flux of solar radiation in cumulus clouds is estimated by comparing its results with the Monte Carlo calculations. The approximate method is based on solving the stochastic transfer equation in a small—angle approximation and sibsequent averaging of the obtained solution over the ensemble of realizations of the cloud field.

Recently, the great attention is paid to the problem of interaction of the radiation with mesoscale cumulus clouds. The most advanced theoretical approach to this problem is based on an approximation of the optical models of cumulus clouds by random indicator fields, which are constructed with the help of the Poisson point processes. By means of averaging the stochastic radiation transfer equation over the ensemble of realizations of a Poisson cloud field, the closed systems of equations have been obtained for the first and second moment of the radiation intensity and algorithms of statistical simulation for calculating the linear functionals of these moments of intensity have been constructed.[1,2] The use of the Monte Carlo method makes it possible to investigate closely the statistical characteristics of the radiation with realistic parameters of the cloud field and to take into account correctly the effects of the multiple scattering. For this reason we may consider the calculations performed according to the algorithms of the Monte Carlo method to be reference and employ them for estimating the accuracy and the limits of applicability of the approximate methods of the statistical description of the radiation transfer in the cumulus clouds.

The purpose of this paper is to estimate the accuracy of one of these approximate methods based on solving the stochastic transfer equation in a small—angle approximation and on subsequent averaging of the solution that has been obtained over the ensemble of realizations of the cloud field.[3,4]

Let a cloud field occupies the layer $0 \le z \le H$. The coefficients of extinction $\sigma(r)$ and scattering $\sigma_{sc}(r)$, and the scattering phase function $g(r;\ \mu)$ at an angle μ are the random scalar fields, namely, $\sigma(r) = \sigma\varkappa(r)$, $\sigma_s(r) = \sigma_s\varkappa(r)$ and $g(r;\ \mu) = g(\mu)\varkappa(r)$, where $\varkappa(r)$ is the random indicator function which is equal to unity when the point r belongs to the cloud and to zero in the opposite case. The field $\varkappa(r)$ is simulated using the Poisson point processes on the straight lines.[1] This field is statistically homogeneous and anisotropic, with the mean value $\langle\varkappa(r)\rangle = N$ and the exponential correlation function $K_\varkappa(r_1,r_2) = \exp\{-A(\omega)|r_1 - r_2|\}$, where N is the cloud amount, $A(\omega) = (|a|) + |b|)A$, $\omega = (a, b, c) = (r_1 - r_2)/|r_1 - r_2|$, $A = (1.65 \cdot N - 0.5)^2 + 1.04]/D$, and D is the characteristic (average) horizontal cloud size. Here and below the angular brackets denote the ensemble average over the realizations of the cloud field.

Let us assume that a unitary flux of the solar radiation $c_\odot = -\arccos \xi_\odot$, where $\xi_\odot$ is the solar zenith angle, is incident at the top of the cloud layer in the direction $\omega_\odot = (a_\odot, b_\odot, c_\odot)$. It follows from the stochastic radiation transfer equation that the random flux $S(r)$ of unscattered light is equal to

$$S(r) = \exp(-\tau(r;\ \sigma)), \qquad (1)$$

where

$$\tau(r;\ \sigma) = \frac{\sigma}{c_\odot}\int_z^H \varkappa(r)d\xi,\quad r' = r + \frac{(\xi - z)}{|c_\odot|}\,\omega_\odot.$$

In the small—angle approximation for the flux of the total transmitted radiation $Q(r)$, the following formula have been obtained in Ref. (3)

$$Q(r) = \exp\,(\tau\,(r,\ \sigma_*)). \qquad (2)$$

where

$$\tau\,(r,\ \sigma_*) = \frac{\sigma_*}{c_\odot}\int_z^H \varkappa(r)d\xi,\quad \sigma_* = \sigma_a + G_0\sigma_{SC}$$

is the effective extinction coefficient, σ_a is the true absorption coefficient, and $G_0 = \int_{-1}^{0} g(\mu)\,d\mu$. According to Eqs. (1) and (2), the flux of the diffuse transmitted radiation $Q_d(r)$ is determined by the expression

$$Q_d = \exp\,(-\tau\,(r,\ \sigma_*)) - \exp\,(-\tau\,(r,\ \sigma)). \qquad (3)$$

From Eqs. (1)–(3) for the mean flux and its variance we obtain

$$\langle S(r)\rangle = \langle \exp\,(-\tau\,(r,\ \sigma))\rangle,$$

and

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$D_s(r) = < \exp(-2\tau(r, \sigma)) > - < S(r) >^2,$

$Q(r) = < \exp(-\tau(r; \sigma_*)) >$

$D_Q(r) = < \exp(-2\tau(r; \sigma)) > - < Q(r) >^2,$

$< Q_d(r) > = < \exp(-\tau(r; \sigma_*)) > - < \exp(-\tau(r; \sigma)) >,$

and

$$D_{Q_d}(r) = D_Q(r) + D_S(r) -$$

$$- 2\big(< \exp(-\tau(r; \sigma_*)) - \exp(-\tau(r; \sigma)) > -$$

$$- < \exp(-\tau(r; \sigma_*)) >< \exp(-\tau(r; \sigma)) >\big). \quad (4)$$

Since the cloud field is statistically homogeneous and the boundary conditions are uniform, the moments of the fluxes depend only on z. The calculation of the mean flux and of the variance of the fluxes Is connected with determination of the function $< j(z) > = < \exp\{-\tau(r; \Sigma)\} >$, where Σ assumes the values σ, 2σ, σ_*, $2\sigma_*$, and $\sigma + \sigma_*$ depending on the considered characteristic. In the framework of the considered mathematical model of the cumulus cloud field, this function is calculated according to the formula[1]

$$< j(z) > = \sum_{i=1}^{2} C_1 \exp\left(-\lambda_1 \frac{(H-z)}{|C_\odot|}\right),$$

where

$$\lambda_{1,2} = \frac{\Sigma + A(\omega)}{2} \pm \frac{\sqrt{(\Sigma + A(\omega))^2 - 4A(\omega)N\Sigma}}{2},$$

$$C_1 = \frac{\lambda_2 - \Sigma N}{\lambda_2 - \lambda_1}, \text{ and } C_2 = 1 - C_1. \quad (5)$$

It is not difficult to show that the spatial correlation functions of the fluxes are expressed in terms of $< j(r_1; \Sigma)\, j(r_2; \Sigma) >$. In the case in which r_1 and r_2 lie in one plane a closed system equations for the correlation $< j(r_1; \Sigma)\, j(r_2; \Sigma) >$ has been obtained from the stochastic radiation transfer equation and solved.[5] Note that the statistical characteristics of the fluxes are invariant under two parameters, namely, the optical depth $\tau = \Sigma H$ and the ratio $\gamma = H/D$.

Let us compare the mathematical expectations, variances, and correlation functions of the fluxes calculated by the first method based on solving the equations for the moments of the intensity by the Monte Carlo method and by the second method based on averaging (over the ensemble of realizations of the cloud field) the solution of the stochastic transfer equation in the small–angle approximation.[3,4] The statistical characteristics of the unscattered radiation coincide.

When providing a physical foundation for the second method, it is assumed that in order to employ it, it is sufficient to satisfy the following condition: the width of Green's function L of the stochastic transfer equation is much less than the horizontal cloud size D. Consequently, with increase of γ (H is fixed and D decreases), the absolute deviation $\Delta = |< Q_d^{(1)} > - < Q_d^{(2)} >|$ is to increase, which is confirmed by the results of calculations shown in Fig. 1. Here the superscript $i = 1, 2$ denotes the mean flux calculated by the corresponding method. It can be seen that the relative error $\delta = \Delta \times 100\% / < Q_d^{(1)} >$ strongly depends on the geometric–optics parameters of the field of cumulus clouds and on the conditions of illumination and can amount to 50–70% (curves 2 for $\gamma = 0$). When the optical depths are small, the second method overestimates the mean flux of the diffuse radiation and conversely underestimates it at large τ.

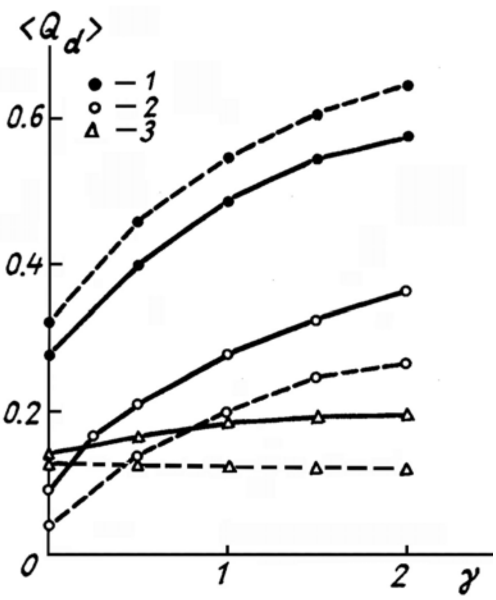

FIG. 1. The mean flux of scattered transmitted radiation as a function of γ at H = 0.5 km for N = 0.5. $\xi_\odot$ = 60°: τ = 5 (1) and 30 (2). $\xi_\odot$ = 0° τ = 30 (3). Here and in Figs. 2, 3, and 4 the solid curves illustrate the first method while the dashed curves illustrate the second method.

Factors random in nature associated with the occurrence of a large number of clouds with finite horizontal sizes in a cloud field influence on the formation of the radiative regime and the brightness fields of cumulus clouds, namely, a possibility for the radiation to enter into and to exit from the clouds through their side surfaces, screening of the incident solar radiation by the surrounding clouds, mutual shadowing, and multiple scattering of light in the gaps between the clouds (radiative interaction of the clouds). These effects account for variability of $< Q_d >$ attendant to changes in γ and with all the other parameters of the problem unchanged. If the Sun is close to the zenith, the second method makes it impossible to take this variability into consideration (Fig. 1, curve *3*).

The analysis of our results shows that the second method has a sufficiently high accuracy for small cloud amounts, namely, when the value of Δ does not exceed 0.02–0.03 in the interval $0 \le \gamma \le 2$ (Fig. 2). For moderate and especially for large cloud amounts when the screening, mutual shadowing, and radiative interaction of the clouds are important for the formation of the radiative fluxes, $< Q_d^{(2)} >$ can be substantially different from $< Q_d^{(1)} >$, and the value of Δ amounts to 0.10–0.15. This circumstance makes it possible to conclude that a possibility to neglect the interaction of the radiative fields of the individual clouds is additional and most important condition of the applicability of the second method. This interaction is negligible when the cloud amounts are small and also in the case of optically thin clouds, for example, cirrus clouds.

In order to calculate the mean radiative fluxes in both cumulus clouds ($\gamma \sim 1$) and stratus clouds ($\gamma \ll 1$), which partially cover the sky, the following formulas are widely used:

$$< S > = 1 - N, \; < Q_d > = NQ_{d,0}, \text{ and } < R > = NR_0, \qquad (6)$$

where $Q_{d,0}$ and R_0 are the diffuse transmission and the albedo of a continuous homogeneous cloud layer. It Is shown[7] that in the limiting case $\gamma \to 0$ formulas (6) can be obtained from the equations for the mean intensity and they describe the mean radiative fluxes in stratus clouds fairly well. The results given in Figs. 1 and 2 testify to the fact that in the case in which the cloud amounts are small and moderate, an employment of asymptotic formulas (6) in order to estimate the mean fluxes in cumulus clouds results in greater errors than that for the second method.

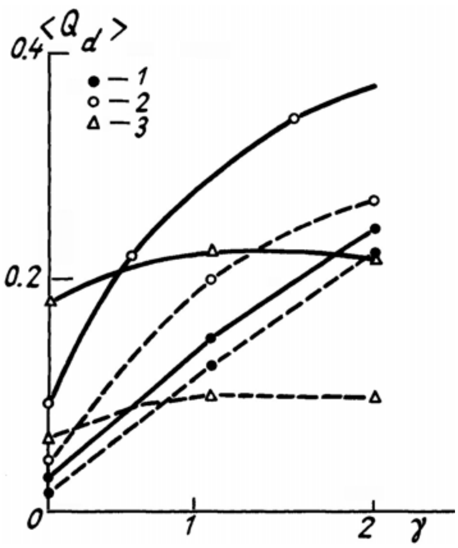

FIG. 2. The mean flux of scattered radiation versus γ *at* $H = 0.5$ km *and* $\tau = 30$ *for* $\xi_\odot = 60°$ *and* $N = 0.1$ *(1),* 0.5 *(2), and* 0.9 *(3).*

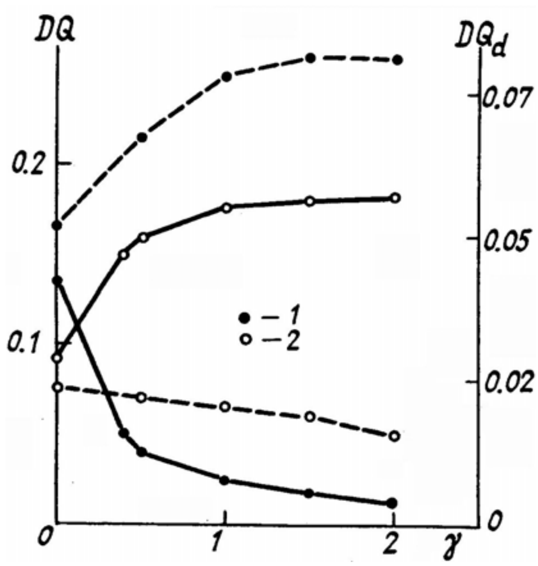

FIG. 3. The variances of the fluxes of transmitted radiation at $H = 0.5$ *km and* $\tau = 15$ *for* $\xi_\odot = 30°$, *and* $N = 0.5$: *the scattered radiation (1) and the total radiation (2).*

The dependence of the variances of the fluxes of the total radiation D_Q and the scattered transmitted radiation D_Q on γ is illustrated in Fig. 3. It can be seen that with decrease of γ (D increases), the accuracy of the second approximate method grows. In the interval $0.5 \le \gamma \le 2$ typical of the cumulus clouds, $D_{Q_d}^{(2)}$ exceeds $D_{Q_d}^{(1)}$ by approximately an order of magnitude, while $D_{Q_d}^{(2)}$ is less than $D_Q^{(1)}$ by a factor of 2–3. It is pertinent to note that $D_Q^{(2)}$ and $D_{Q_d}^{(2)}$ almost coincide, which contradicts the available experimental data presented in Ref. 8, namely, the variance of the flux of the total radiation exceeds the variance of the flux of unscattered transmitted radiation by a factor of ~ 2.

The correlation function $K_Q(L)$ of the flux of the total radiation as a function of γ is plotted in Fig. 4, where $L = |r_1 - r_2|$, $r_1 = (x_1, 0, 0)$ and $i = 1, 2$. It is well known[8] that $K_Q(L)$ is determined to a considerable extent by the correlation of the flux of unscattered light, which is calculated from the same analytical formulas in both cases. This fact explains a satisfactory agreement (especially when the values of γ are small) between $K_Q^{(1)}(L)$ and $K_Q^{(2)}(L)$.

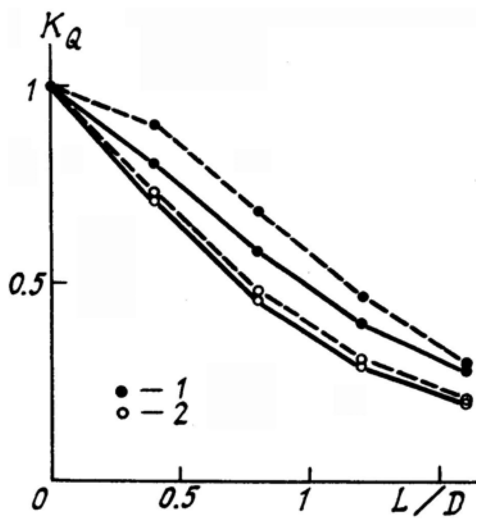

FIG. 4. The correlation functions of the fluxes of the total transmitted radiation at $H = 0.5$ km *and* $\tau = 15$ *for* $\xi_\odot = 30°$, $N = 0.5$, and $\gamma = 2$ *(1) and* 0.5 *(2).*

The analysis performed shows that simple analytical expressions used to calculate the mean fluxes of the short-wave radiation in Refs. 3 and 4 provide a satisfactory accuracy except for the case of large cloud amounts and small solar zenith angles for optically thick clouds. Based on these expressions, it is also possible to calculate the correlation functions with a sufficiently high accuracy, but these expressions yield great errors in calculating the variances of the fluxes of transmitted radiation for optically thick clouds with the parameter $\gamma \ge 1$.

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