# INTENSITY FLUCTUATIONS OF LASER BEAMS IN A TURBULENT ATMOSPHERE

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*The paper presents a survey of the experimental and theoretical results of investigations of laser beam intensity fluctuations in a turbulent atmosphere carried out at the Institute of Atmospheric Optics. Spatial and temporal statistical characteristics of coherent laser beams and partially coherent radiation are considered. The data on laser beam intensity fluctuations in the turbulent atmosphere under conditions of precipitation are also given.*

The study of statistical properties of laser beam intensity in a turbulent atmosphere is a traditional direction of investigations of the Institute of Atmospheric Optics. Since the latter half of the 1960s, first in the Laboratory of Infrared Radiation of the Siberian Physical–Technical Institute and then in the Laboratory of Optics of Randomly Inhomogeneous Media of the Institute of Atmospheric Optics the purposeful experiments have been carried out on atmospheric paths to study the peculiarities of the intensity fluctuations in spatially bounded laser beams. Nowadays these studies are performed mainly in the Laboratories of Wave Diffraction, Nonlinear Optics of Turbulence, and Optics of Turbulent Media.

This paper presents the most important results obtained at the Institute of Atmospheric Optics on the laser radiation intensity fluctuations. We have considered not only spatial and temporal statistical characteristics of coherent beam intensity but also these for partially coherent radiation as well as the peculiarities of turbulent fluctuations of laser beam intensity under fluctuating wind and precipitation conditions. Academician V.E. Zuev, considering this field of investigations as part of the fundamental problem of optical radiation propagation in the atmosphere, has consistently supported the performance of the research. He generalized the results obtained in this field at different stages in his monographs.[1,2]

## VARIANCE OF THE INTENSITY FLUCTUATIONS

The advent of highly coherent laser sources, which make possible long–range transmission of the narrow–beam energy, generated a need for the study of regularities of the intensity fluctuations determined by the influence of diffraction parameters of wave beams. Another problem urgent at that time was a problem of strong intensity fluctuations in a beam propagating along extended paths due to multiple scattering on the inhomogeneities of a medium. The study of strong fluctuations attendant to radiation focusing, when the radius of phase front curvature $F$ in the center of emitting aperture equals the path length ($F/L = 1$), was one of the "hottest" areas in current research. In this case the regime of saturated scintillations is realized on comparatively short paths about one hundred meters in length. The first measurements of the intensity fluctuations,[3–5] performed at the Institute, were aimed at the solution of these problems.

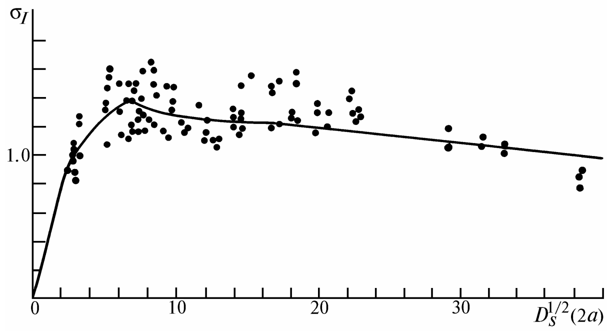


*FIG. 1. Standard deviation of the intensity fluctuations on the axis of a focused laser beam: experiment (points) and calculation in phase approximation of the Huygens–Kirchhoff method (solid line).*

Figure 1 shows the data of measurement of the variance of a focused beam intensity performed in Ref. 4 as function of the parameter $D_s(2\,a) = 2.84\,\beta_0^2\,\Omega^{5/6}$, characterizing the turbulent conditions of propagation along the path (regime of optical turbulence). Here $\Omega = k\,a^2/\,L$ is the Fresnel number of a transmitting aperture of radius $a$, $k$ is the wave number, and $L$ is the path length; $\beta_0^2 = 1.23\,C_n^2\,k^{7/6}\,L^{11/6}$ is the variance of the plane wave intensity, calculated to the first order of the smooth perturbation method,[6] $C_n^2$ is the structure constant of turbulent pulsations of the refractive index of ambient air). The experiments were carried out on the paths of length $L = 20 \ldots 1360$ m using a He–Ne laser radiation at $\lambda = 0.63$ μm with the Fresnel number of a source aperture $\Omega \geq 5$. It is clear that with the increase of the parameter $D_s(2\,a)$ (path length or/and structure constant of the refractive index) the relative variance of laser radiation intensity

$$\sigma_I^2 = \frac{<I^2>}{<I>^2} - 1 \qquad (1)$$

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first increases and then saturates at a level close to unity. In the vicinity of $D_s(2a) \approx 50$, the focus of the intensity fluctuations occurs, where the value $\sigma_I$ is at its maximum.

Figure 2 vividly illustrates the dependence of the intensity variance on the axis of a collimated beam on the Fresnel number of a transmitting aperture. Here empty circles indicate the experimental data for the regime of weak fluctuations obtained at the Institute of Atmospheric Optics.[3] We have found that the transfer to the regime of spatially bounded beams is accompanied by the decrease of the intensity fluctuations as compared to unbounded plane and spherical waves. In the narrowest collimated beams ($\Omega \approx 1$) the variance is minimal.

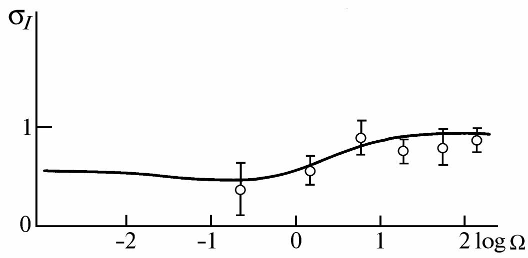



*FIG. 2. Dependence of $\sigma_I$ on the Fresnel number of a transmitting aperture. Solid curve shows the calculation by the formulas derived in Ref. 2 to the first order of the smooth perturbation method.*

## PHASE APPROXIMATION OF THE HUYGENS–KIRCHHOFF METHOD (PAHKM)

The Rytov smooth perturbation method[6,7] remained the main operating method of the theory of propagation of optical waves in a turbulent atmosphere until the early 1970s. However, the applicability of this method is limited by the conditions of weak intensity fluctuations, realized at comparatively short distances, when the parameter $\beta_0^2$ takes the values not exceeding unity.

An essential advance in the theory of wave propagation in large–scale randomly inhomogeneous media is based on Refs. 8–12, where the equations were obtained for statistical moments of the field complex amplitude, adequately describing the intensity statistics under arbitrary propagation conditions including the regime of strong fluctuations. Unfortunately, a solution of the equation for the fourth moment, directly describing the intensity fluctuations, is represented by the continual Feynman integral,[13,14] whose calculation by numerical methods is very difficult because of oscillating character of the approximating integrand. Efficient algorithms for calculating this integral have recently appeared,[15,16] and in the early 1970s the asymptotic and approximate solutions of the equations for the fourth moment were obtained in some specific cases only.[17–19]

Much progress has been made in studying the intensity fluctuations in spatially bounded laser beams owing to the use of the phase approximation of the Huygens–Kirchhoff method (PAHKM) developed at the Institute of Atmospheric Optics.[20–22] Let us represent the complex amplitude of wave field $U(x, \rho)$ at the point $\rho\{y, z\}$ of the plane of observation $x' = x$ in terms of the Green's function of the wave parabolic equation $G(x, x_0; \rho, \mathbf{t})$:

$$U(x,\rho) = \int \mathrm{d}\mathbf{t}\, U_0(\mathbf{t})\, G(x, x_0; \rho, \mathbf{t}) , \tag{2}$$

where $U_0(\mathbf{t}) = U(x_0, \mathbf{t})$ is the initial field distribution in the plane $x' = x_0$, $G(x_0, x_0; \rho, \mathbf{t}) = \delta(\rho - \mathbf{t})$, and $\delta(\mathbf{z})$ is the Dirac delta function. The Green's function in the randomly inhomogeneous medium can be written as

$$G(x, x_0; \rho, \mathbf{t}) = \\ = \frac{k \exp[i\,k(x - x_0)]}{2\pi i(x - x_0)} \exp\left\{ i\kappa \frac{(\rho - \mathbf{t})^2}{2(x - x_0)} + \tilde{\psi}(x, x_0; \rho, \mathbf{t}) \right\}, \tag{3}$$

where the function $\tilde{\psi}$ specifies the random run–on of the complex phase of a spherical wave propagating from the point $(x_0, \mathbf{t})$ to the point $(x, \rho)$.

On the assumption that phase relations between the fields of elementary spherical waves from different points of a transmitting aperture, that interfere at the observation point, play a leading part as compared with the amplitude ones, we proposed to use the "phase" approximation[20]

$$\tilde{\psi}(x, x_0; \rho, \rho') = i\,\tilde{S}(x, x_0; \rho, \rho'), \tag{4}$$

and the calculation of the random phase $\tilde{S}(x, x_0; \rho, \rho')$ was proposed to be made to the first order of the geometrical optics method[6,7]:

$$\tilde{S}(x, x_0; \rho, \rho') = \frac{k}{2} \int_{S_0}^{S} \varepsilon_1(\mathbf{m}(x, x_0; \rho, \rho'),\zeta)\, \mathrm{d}\zeta =$$

$$= \frac{k}{2} \int_{x_0}^{x} \varepsilon_1\left(x', \rho \frac{x' - x_0}{x - x_0} + \rho' \frac{x - x'}{x - x_0}\right) \mathrm{d}x' . \tag{5}$$

As shown in Refs. 14, 23, and 24, this approximation is equivalent to replacement of the continual integration over a set of trajectories connecting the end points of the path in the Feynman integral representation of a wave field by the straight line $l(x') = \rho[(x' - x_0)/(x - x_0)] + \rho'[(x - x')/(x - x_0)]$.

In Refs. 25–27 the mathematical substantiation is given for the PAHKM applicability to calculating the intensity fluctuations of optical radiation in randomly inhomogeneous media. For this purpose in the above–indicated papers the differential equation for the fourth field moment was transformed into the integral equation in such a way that the latter contained the expression for the $\Gamma_4^k$, calculated in the phase approximation of the Huygens–Kirchhoff method

$$\Gamma_4^k(x, \underline{\rho_4}) = \int \Gamma_4(x_0, \underline{\rho'_4})\, G_4^k(x, x_0; \underline{\rho_4}, \underline{\rho'_4})\, \mathrm{d}\underline{\rho'_4} , \tag{6}$$

as a free term, where

$$G_4^k(x, x_0; \underline{\rho_4}, \underline{\rho'_4}) = \left(\frac{k}{2\pi(x - x_0)}\right)^4 \times$$

$$\times \exp\left\{ i \frac{k}{2(x - x_0)} \sum_{j=1}^{4} (-1)^{j-1} (\rho_j - \rho'_j)^2 \right\} \times$$

$$\times \left\langle \exp\left\{ i \sum_{j=1}^{4} (-1)^{j-1} \tilde{S}(x, x_0; \rho_j, \rho'_j) \right\} \right\rangle , \tag{7}$$

$\rho_{\underline{4}} = \{\rho_1, \rho_2, \rho_3, \rho_4\}$, $\underline{4}$ means the change of the subscript from 1 to 4.

As a result of analysis of the Neumann series of the integral equation so constructed, we have managed to elucidate that the PAHKM adequately describes the intensity fluctuations in beams focused by the apertures whose size $2a$ satisfies the condition $\Omega \gg \beta_0^{84/25}$ for $\beta_0^2 \gg 1$ and $\Omega \gg 1$ for $\beta_0^2 < 1$. The condition of applicability of this approximation to the beams focused by the apertures of less size and collimated beams is the requirement that the size of transmitting aperture of a source was no less than the radius of the first Fresnel zone $\Omega \geq 1$. The merit of the PAHKM is the fact that this method makes it possible to investigate the laser radiation field fluctuations by simple numerical methods for arbitrary values of the parameters $D_s(2a)$ and $\beta_0^2$, characterizing turbulent conditions of propagation.

From Eq. (7), in particular, for the relative variance of the focused beam intensity we have[2,22]

$$\sigma_I^2 = \Big[ \frac{1}{2}(\Gamma^2(11/6) + \Gamma(8/3)) -$$

$$- \Gamma^2(11/6)\,(3/4)^{8/3}\,{}_2F_1(11/6,\ 11/6;\ 1;\ 1/4)\Big]\, D_s^2(2a) \quad (8)$$

for the conditions of weak intensity fluctuations ($D_s(2a) < 1$) and

$$\sigma_I^2 = 1 + 2.7\,\Omega^{-1/3}\,\beta_0^{-4/5} =$$

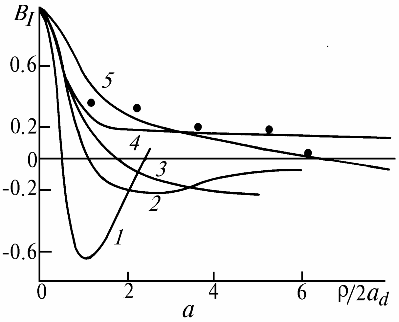
$$= 1 + 4.1\,D_s^{-2/5}(2a) + 0(D_s^{-4/5}(2a)) \quad (9)$$

in the case of strong fluctuations[29] ($D_s(2a) >> 1$). From Eqs. (8) and (9) it follows that with the increase of the strength of optical turbulence (parameter $D_s(2a)$) relative fluctuations in a focused beam, in absolute agreement with the experimental data, first increase and then saturate at a constant level being equal to unity. The results of numerical calculation of the $\sigma_I$ dependence on the parameter $D_s(2a)$ in the phase approximation for a focused beam are shown in Fig. 1. It is clear that the calculation agrees well with the experimental data obtained in Ref. 4.

At the Institute of Atmospheric Optics some other methods of approximation for solving the stochastic equation[28] have also been developed, among them are the phase approximation of the spectral expansion method (PASEM) and the spectral phase approximation of the Huygens–Kirchhoff method (SPAHKM). In PASEM we use the expansion of the unknown field in the spectrum of plane waves, and the field of its applicability for calculating the intensity fluctuations is limited by the condition $\Omega \leq 1$. In the PAHKM we perform successive expansion of the field in spherical and plane waves. This makes it possible to obtain uniform convergence of solution of the parabolic equation in a random medium not only in the parameter of turbulence $\beta_0^2$, as in PAHKM and PASEM, but also in the diffraction parameter $\Omega$. As PAHKM, the above–mentioned approximations consist in replacement of continuous integration in a rigorous expression for the field in the form of the Feynmann integral by integrating along the only properly chosen path.[23]

## SPATIAL CORRELATION AND TEMPORAL SPECTRA OF THE INTENSITY FLUCTUATIONS

The application of PAHKM in the investigations into the intensity fluctuations has made it possible to determine a number of novel physical regularities in the behavior of the scales of intensity correlation in laser beams attendant to changes in the turbulent conditions of propagation.



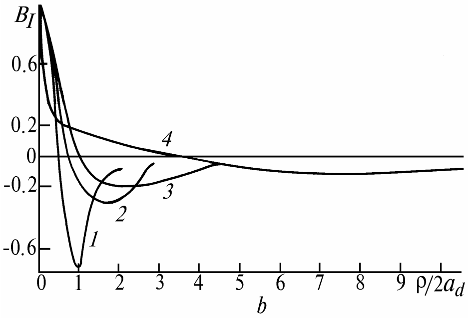




*FIG. 3. Coefficient of the spatial intensity correlation: a) focused beam* ($L/F = 1$, $\Omega = 25$), $D_s(2a) = 2$ (*1*), 10 (*2*), 50 (*3*), 400 (*4*), *and* 1400 (*5*), *dots denote the experiment from Ref. 30* ($\Omega = 8.7$); *b) collimated beam* ($L/F = 0$, $\Omega = 1$), $D_s(2a) = 3$ (*1*), 16 (*2*), 32 (*3*), *and* 158 (*4*).

The results of calculation of the spatial correlation coefficient for symmetric separation of observation points from the beam axis are shown in Fig. 3. The values of the parameter $D_s(2a)$ are chosen in such a way that to consider the weak intensity fluctuations, the region of fluctuation focusing (see Fig. 1), and the interval of asymptotic saturation of the intensity variance. It follows from the shape of the curves that the radius of positive correlation of weak intensity fluctuations approximately equals the beam diffraction radius. When separating the observation points at the distance of the order of beam diffraction diameter, profound negative correlation is observed. This pattern of spatial correlation for weak intensity fluctuations agrees well with the results of calculations performed in Ref. 28 to the first order of the smooth perturbation method. When going from weak fluctuations to the regime of variance saturation ($D_s^{1/2}(2a) > 10$), the shape of the correlation function changes. In the case of radiation focusing (Fig. 3*a*), the radius of positive correlation remains approximately equal to the beam diffraction diameter $2a_d = 2L/ka$, that coincides with the results of Refs. 18 and 19. Simultaneously the second scale of positive (residual) correlation appears which exceeds significantly the first scale. The experimental data from Ref. 30 are also shown here. They agree well with the calculation when separation $\rho > 2a_d$.

As differentiated from the focused beam, the radius of positive correlation of a collimated beam, in going to the regime of variance saturation (curve *4* in Fig. 3*b*) becomes much less than the diffraction radius. As in the case of radiation focusing, the second scale of correlation appears.

Having determined the radius $r_I$ of spatial correlation of the intensity from the condition of the decrease of the correlation coefficient down to the level $e^{-1}$ and using the phase approximation of the Huygens–Kirchhoff method, for the regime of strong intensity fluctuations we can obtain the formula[2]

$$r_I = 2^{-1/2}\,\rho_c \tag{10}$$

where

$$\rho_c = \rho_p \left( \frac{(1 - L/F)^2 + \Omega^{-2} + 4/3(a_d/\rho_p)^2}{1 - L/F + 1/3(L/F)^2 + 1/3\Omega^{-2} + 4/3(a_d/\rho_p)^2} \right)^{-1/2} \tag{11}$$

is the radius of spatial coherence of the beam field, $\rho_p = 0.9\sqrt{L/k}\,\beta_0^{-6/5}$ is the radius of coherence of a plane wave. It follows from Eqs. (10) and (11) that in the regime of strong fluctuations for spatially bounded collimated beams ($L/F = 0$, $\beta_0^{-12/5} \le \Omega \le \beta_0^{12/5}$) the correlation radius is at its maximum $r_I = 2^{1/2}\rho_p$, in the case of unbounded plane wave ($\Omega \gg \beta_0^{12/5}$) $r_I = 2^{-1/2}\rho_p$, and in the case of a spherical wave ($\Omega \ll \beta_0^{-12/5}$) $r_I = \sqrt{3/2}\,\rho_p$. Equation (10) also remains valid when the radiation is focused to the observation plane ($L/F = 1$), if the condition $\Omega \ll \beta_0^{12/5}$ is fulfilled. In the opposite case $\Omega \gg \beta_0^{12/5}$, as was mentioned above, the radius of spatial correlation is determined by the beam radius in a homogeneous medium $r_I = 2^{-1/2} a_d$.

The effect of the increase in the scale of spatial correlation for strong intensity fluctuations in spatially bounded beams has been experimentally confirmed.[31–34] Figure 4 shows the measurements of the coefficient of spatial intensity correlation in narrow collimated beams ($\Omega = 1$, $\Omega = 2.2$) and in plane and spherical waves. It is clear that at equal separations of observation points and for close values of the turbulence parameter $\beta_0^2$ the degree of the intensity correlation in narrow beams is higher than that in unbounded waves. The calculation carried out using the PAHKM for the experimental conditions of Ref. 31 is in good agreement with the measurements.

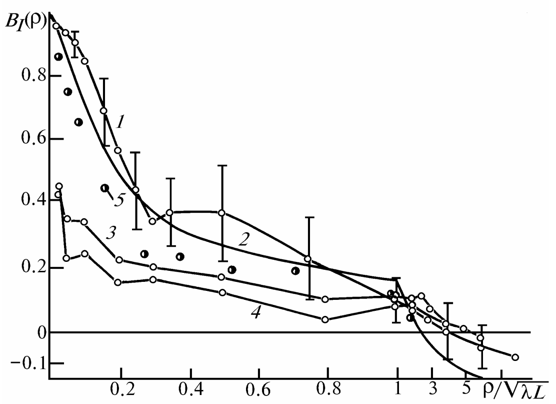



*FIG. 4. Spatial correlation coefficient for strong intensity fluctuations: 1) experiment from Ref. 31* ($\beta_0 = 3.3 \ldots 6.1$ *and* $\Omega = 1$); *2) calculation by PAHKM* ($\beta_0 = 7.3$ *and* $\Omega = 1$); *3–5) experimental data from Refs. 32–34 for spherical and plane waves and narrow beam* ($\Omega = 2.2$).

The use of a hypothesis that the turbulence of the air refractive index in the atmosphere be frozen, that is, $n(\mathbf{r}, t + \tau) = n(\mathbf{r} - \mathbf{V}\tau, t)$, where $\mathbf{V}$ is the drift velocity of the turbulent inhomogeneities of a medium, enables one to analyze the temporal correlation functions and spectra of laser beam intensity fluctuations. A detailed analysis of temporal characteristics of the intensity fluctuations was performed in the limits of applicability of the first order of the smooth perturbation method in Refs. 6 and 28, where it was found that the typical temporal scale for weak intensity fluctuations is the wind–drift time of the inhomogeneities of a medium across the source aperture $\tau_0 = 2a/V_\perp$ ($V_\perp$ is the wind velocity component orthogonal to the path). The use of the PAHKM allowed calculations to be done for strong intensity fluctuations.[2,22,35]

In particular, it was found that the typical scale of temporal intensity correlation defined as the fall time $\tau_c$ of the correlation coefficient down to the level $e^{-1}$ for $\beta_0^2 \gg 1$, in contrast to the radius of spatial correlation $r_I$, is the same for both plane and spherical waves

$$\tau_c \equiv \tau_{pc} = \rho_p / (\sqrt{2}\,V_\perp) \tag{12}$$

and in the regime of a space–bounded beam becomes twice as large as that for unbounded waves

$$\tau_b \equiv 2\tau_{pc} = \sqrt{2}\,\rho_p / V_\perp . \tag{13}$$

When radiation is focused, $\tau_c$ is also determined by formula (13) regardless of the size of a focusing aperture; whence it follows that strong intensity fluctuations are characterized by the temporal scale proportional to the wind–drift time of space inhomogeneities of a medium at the distance of the order of the field coherence radius $\rho_p$.

Figure 5 shows the results of calculation of the dimensionless normalized temporal spectra of the intensity fluctuations $U_I(f) = W(f)\,f/\sigma_I^2$ (here $W(f)$ is the frequency spectrum of the intensity fluctuations and $f$ is the frequency, in Hz) under different turbulent conditions of propagation characterized by the parameter $D_s(2a)$. As can be seen from the figure, for small values of $D_s(2a)$ (weak intensity fluctuations) the function $U_I(f)$ has the sharply pronounced maximum at the frequency $f_0 = 1/\tau_0$ (curves *1* and *2* are depicted on the scale $f/f_0$). With the increase of the parameter $D_s(2a)$ the spectrum is broadened, and the maximum of spectral power density is shifted toward higher frequencies. For very large value of $D_s(2a)$ ($D_s(2a) = 1400$) the two–scale nature of temporal fluctuations manifests itself in the spectra. In this case the primary contribution to the intensity fluctuations comes from the frequencies close to $f_c = 1/\tau_c$ (curve *3* is depicted on the scale $f/f_c$). On the same scale the average experimental curve *4* is drawn for $\Omega = 26$ and $625 \le \beta_0^2 \le 900$ (see Ref. 33). The calculation in the phase approximation of the Huygens–Kirchhoff method, although performed at $\Omega = 1$, describes adequately the experimental data since the dependence of the temporal correlation scale $\tau_c$ on the diffraction conditions at the transmitting aperture is rather weak (see Eqs. (12) and (13)).

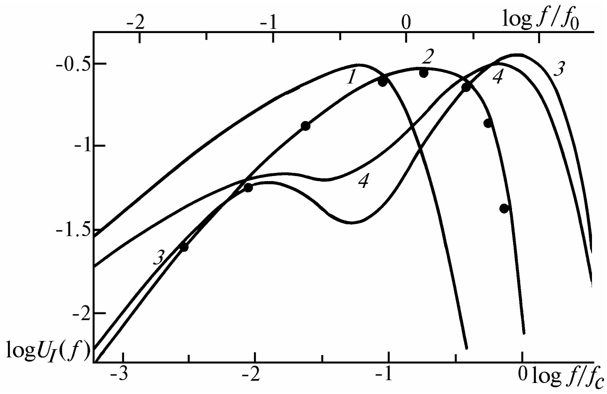



*FIG. 5. The intensity fluctuations spectrum: calculation by PAHKM at $D_s(2a) = 3$ (1), 160 (2), and 1400 (3), $\beta_0^2 = 440$, and $\Omega = 1$; average experimental curve from Ref. 33 for $\Omega = 26$ and $\beta_0^2 = 625 \ldots 900$ (4); dots correspond to the experimental data from Ref. 36 for $\Omega = 1.2$ and $\beta_0^2 = 8$.*

The spectrum measured in Ref. 36 ($\lambda = 10.6$ μm, $L = 14.5$ km) is closest to calculated curve 2 in Fig. 5 in the values of the parameters $\beta_0^2$ and $\Omega$. The Fresnel number of the transmitting aperture in this experiment was $\Omega = 1.2$. A comparison of the experimental data from Ref. 36 with the results of calculation shown in Fig. 5 and performed on the scale $f/f_0$ demonstrates satisfactory agreement between the theory and the experiment.

## SPECTRA OF THE LIGHT WAVE INTENSITY IN FLUCTUATING WIND

At the Institute of Atmospheric Optics much attention has been given to the study of the regularities determining the behavior of temporal characteristics of laser radiation intensity fluctuations when the fluctuation component of wind velocity is comparable to or in excess of its average value. Until the work done at the Institute of Atmospheric Optics, only some references concerning separate measurements of this kind received individual mention in the scientific literature, while on the paths aligned with the mean wind velocity direction or close to it the influence of the turbulent velocity fluctuations on temporal characteristics of the intensity becomes the governing factor. As a result of analysis of the extensive experimental material obtained at the Institute,[37–41] it has managed to find the parameter of similarity enabling one to systematize the measurements of the temporal intensity fluctuations for any relation between the mean and the fluctuation components of the wind velocity.

To elucidate the above–indicated regularities, the measurements of the temporal correlation functions and frequency spectra of the intensity were carried out on mutually perpendicular paths,[38] one of which was oriented in the transverse direction and another – along the wind direction. The wind velocity data were synchronously recorded. In different measurement runs the mean of the transverse component of the wind velocity $V_{\perp}$ varied within (0.1– 4) m/s and the standard deviation of the velocity was $\sigma_{\perp} = (0.2–0.7)$ m/s. Thus in the experiments the ratio $\sigma_{\perp}/V_{\perp}$ varied within the limits 0.1–10.

Figure 6 shows the results of synchronous measurements of the frequency spectra of the intensity fluctuations of quasispherical waves with the Fresnel number of transmitting aperture $\Omega = 10^{-2}$ propagating along perpendicular paths. The dimensionless spectrum $U_I$ is plotted on the $Y$ axis at the left, and at the right – the logarithm of the dimensionless spectrum is plotted. The representation of the spectrum on the semilogarithmic scale shows the contribution of fluctuations at different frequencies to the variance $s_I^2$.

Figure 6 *a* introduces the dimensionless frequency $f/f_0$ obtained by normalization to the usually used parameter $f_0 = V_{\perp}/\sqrt{2\pi\lambda L}$ depending only on the mean wind velocity in the plane transverse to the path. It is clear from the figure that the temporal spectra of the spherical wave intensity fluctuations along the two perpendicular paths, depicted on so defined scale, appear to be frequency–diversed. This is due to the fact that the intensity fluctuations for $\sigma_{\perp}/V_{\perp} > 1$ (curves 2 and 2′) are determined not only by small drift of the refractive index inhomogeneities across the path but also to a greater extent by their much more intensive mixing.

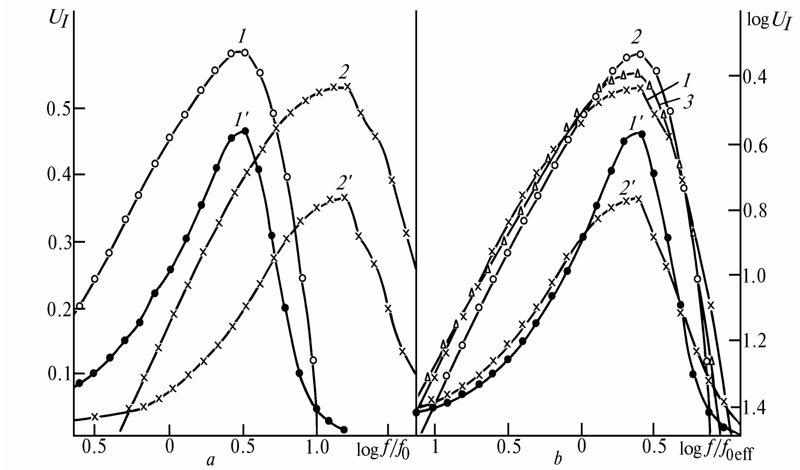



*FIG. 6. Change in the position of frequency spectra after introducing the dimensionless parameters $f/f_0$ and $f/f_{0\mathrm{eff}}$:* $V = 2.2$ m/s, $\sigma_{\perp}/V_{\perp} = 0.2$ (1); $V_{\perp} = 0.1$ m/s, $\sigma_{\perp} V_{\perp}/V_{\perp} = 3 \ldots 4$ (2); *the same spectra on semilogarithmic scale* (1′ and 2′); $V_{\perp} = 1.4$ m/s, $\sigma_{\perp}/V_{\perp} = 0.5$ (3).

Therefore, for universal description of temporal characteristics of the intensity fluctuations in Ref. 38 it was proposed to use the frequency normalization parameter considering the fluctuation component of the wind velocity. Introduction of the dimensionless effective frequency $f_{0\mathrm{eff}} = \sqrt{(V_\perp^2 + \sigma_\perp^2)/(2\pi\lambda L)}$ (Fig. 6 *b*) allows the estimates of the spectra along these paths to approach each other. For $\sigma_\perp \geq 0.8\ V_\perp$ the shape and position of the spectra on the axis of dimensionless frequency $f/f_{0\mathrm{eff}}$ remain practically invariant.

In the experiments most measured spectra lie between the dependence $\sigma_\perp > V_\perp$ (small crosses) and $\sigma_\perp = 0.15 V_\perp$ (points). The fluctuation component of wind velocity less than $0.15 V_\perp$ was observed in none of the experiments. Therefore, for correct evaluation of the effective frequencies, at which the turbulent intensity fluctuations attain their maxima, it is important to take into account not only the mean but also the fluctuation components of wind velocity. This can be made, in particular, by introducing the parameter of similarity $f_{0\mathrm{eff}}$ (see Ref. 38).

## PROBABILITY DENSITY OF THE INTENSITY FLUCTUATIONS

The probability density of the intensity fluctuations is the most complete single–point statistical characteristics, which enables one to evaluate reliability and noise stability of optical communication lines, noise of goniometric and range–finding optical systems, and so on. The interest in this problem, in addition to its practical importance, is connected with the fact that adequate theoretical model describing the probability density of the optical radiation intensity in the whole range of variation of turbulent conditions of propagation, has not yet been developed.

At the Institute of Atmospheric Optics the extensive experimental investigations into the probability density of the intensity have been made on both one–way paths and paths with reflection for a wide variety of turbulent conditions of propagation.[42,43] The obtained experimental data have confirmed the applicability of the model of lognormal intensity distribution under conditions of weak fluctuations ($\beta_0^2 < 1$).[44] However, it is shown that the extension of this model to the regime of strong fluctuations[44] is not valid. A comparative analysis of experimental data and model distributions indicates that the *K*–distribution describes the saturated fluctuations ($\beta_0^2 \geq 100$) on one–way paths as well as approximates the experimental data on reflection of a spherical wave from the matrix of corner–cube reflectors under conditions of strong fluctuations asymptotically more adequately than lognormal distribution.

A comparison of the histogram of saturated intensity fluctuations of a plane wave with the model *K*–distribution is shown in Fig.7 (see Ref. 43). The probability density $P(I)$ determined by this distribution in the range $0.1 < I/\langle I\rangle < 2.5$ is closest to the histogram. In the deep fading range $0.01 < I/\langle I\rangle < 0.025$ the excess of the model histogram is 2–2.5 orders of magnitude, in intermediate fading range $0.02 < I/\langle I\rangle < 0.1$ the difference is 0.5–1 order of magnitude, and in the range of overshoots $I/\langle I\rangle \geq 10$ the difference is about 3 orders of magnitude.

Figure 8 shows good agreement of the *K*–distribution with the experimental data in the case of spherical wave reflection from a matrix of 12 closely–packed corner–cube reflectors.[45] In this case the probability density remains practically within the statistical spread under conditions of saturated intensity fluctuations ($\beta_0^2 > 10$).

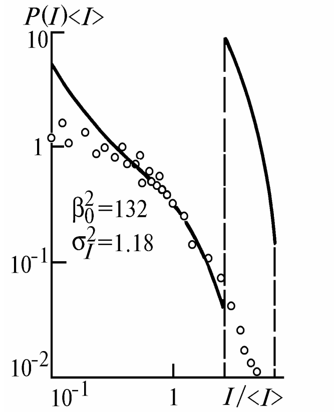



*FIG. 7. Comparison of the histogram (circles) of the normalized intensity for a plane wave with the K–distribution model (solid lines).*

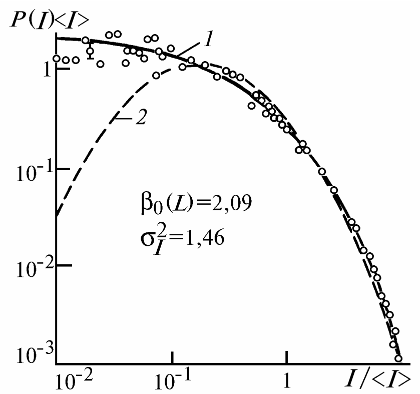



*FIG. 8. Comparison of the histogram of the normalized intensity with the K–distribution (1) and lognormal distribution (2) for a spherical wave reflected from a matrix of 12 corner cubes.*

A detailed analysis of the results obtained at the Institute of Atmospheric Optics in the field of investigation into the probability density of laser radiation intensity can be found in Ref. 43, where the errors in determining higher normalized moments are analyzed along with the laws of distribution. The influence of dynamic range of the instruments on the accuracy of the estimate of the intensity probability density is also investigated there.

## LASER RADIATION INTENSITY FLUCTUATIONS IN THE TURBULENT ATMOSPHERE UNDER CONDITIONS OF PRECIPITATION

An original field, which has been developed at the Institute of Atmospheric Optics for many years, is the investigation of the laser radiation intensity fluctuations in the turbulent atmosphere under conditions of precipitation. Statistics of the intensity in this case is determined by both continuous turbulent inhomogeneities of the refractive index and discrete ones engendered by the precipitation particles.[46–50] Under conditions of precipitation turbulent mixing of air becomes less intensive, and the radiation scattering by precipitation particles becomes dominating factor resulting in essential differences in the behavior of statistical characteristics of the intensity as compared with the propagation through the turbulent atmosphere.

Figure 9 shows the data on measurements of the variance of the laser beam intensity in snowfalls depending on the optical thickness $\tau$ on the paths 130–1930 m long.[46] The data shown in Fig. 9*a* indicate that with the increase of optical thickness of precipitation the radiation intensity fluctuations first grow larger and then saturate at a certain level in the range of optical thicknesses $0.6 \le \tau \le 4$. In contrast to the propagation through the turbulent atmosphere, the saturation level of the intensity variance in this case appears to be less than unity. It is determined by the size of particles in snowfalls. Further increase of optical thickness leads to vanishing of the intensity fluctuations, as follows from Fig. 9*b*.

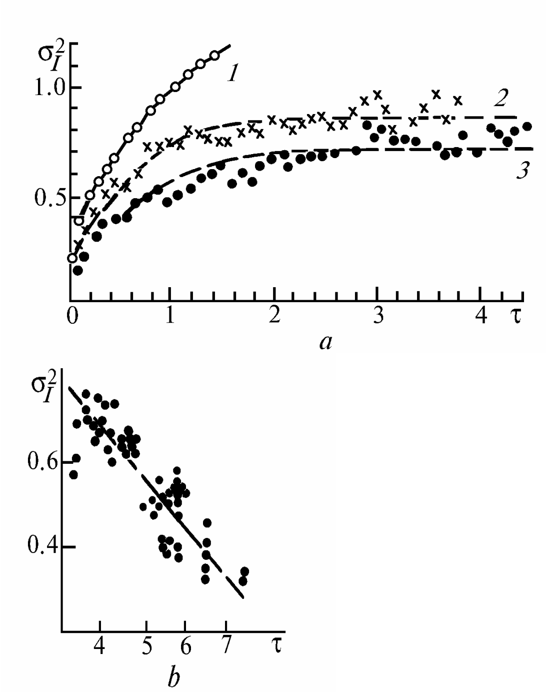



*FIG. 9. Variance of the laser radiation intensity in snowfalls for different maximum size of particles $d_{max}$: a) $d_{max}$ = 7 (1), 3 ... 5 (2), and 1 ... 3 mm (3), b) $d_{max}$ = 1 ... 3 mm.*

The dynamics of transformation of the radiation intensity fluctuation spectra attendant to changes in the precipitation intensity (optical thickness) is illustrated by the experimental data in Fig. 10. It is clear that first the spectrum has more pronounced low–frequency portion determined by the wind drift of the turbulent inhomogeneities (curve *1*). With the increase of $\tau$ it gradually transforms in the spectrum with more pronounced high–frequency portion centered at the frequences determined by the ratio of the particle velocity to their size.[46,48] As the extensive experimental studies have indicated,[46,48] the hydrometeor maximum in the intensity spectra can be found at a frequency as high as 1.6 kHz, while in the atmosphere without precipitation the turbulent intensity fluctuation spectra lie in the low–frequency range, with the cutoff frequency not exceeding 1 kHz.

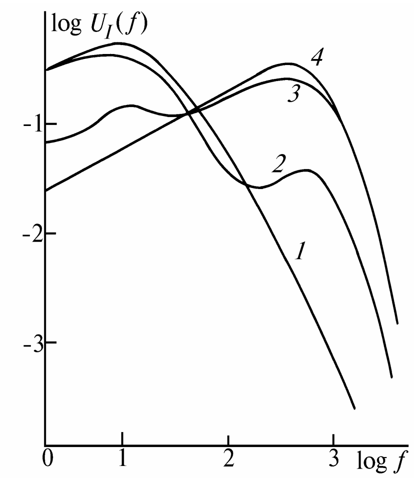



*FIG. 10. Temporal spectra of the intensity fluctuations in a turbulent atmosphere with precipitation particles of the size $d_{max}$ < 5 mm at $\Omega$ = 4.5 and optical thickness $\tau$ = 0 (1), 0.2 (2), 0.6 (3), and 1.2 (4).*

A good example vividly demonstrating the relative role of turbulence and precipitation in the formation of the intensity fluctuations of laser beams is the dependence of relative variance on the focusing parameter $L/F$ (Fig. 11). As is clear from the figure, the three types of dependences can be realized, namely, neutral (curve *1*), with maximum (curve *2*), and with minimum (curve *3*) for the parameter $L/F$ close to unity. Such a set of dependences is explained[46,51] by different contributions from hydrometeor and turbulent components of the refractive index inhomogeneities to the measured variance. If the turbulent contribution $\sigma^2_{I,T}$ prevails, the dependence of type *3* is realized, characteristic of the propagation in the atmosphere without precipitation, and if the precipitation contribution $\sigma^2_{I,P}$ prevails, we observe the dependence of type *2*. For relatively equal contributions from both components, the dependence $\sigma_I$ on the parameter of focusing is neutral.

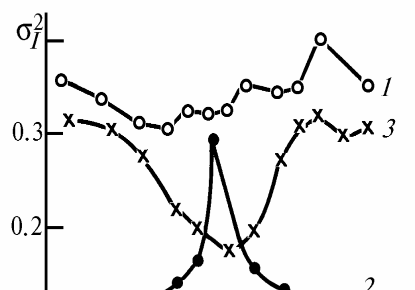



*FIG. 11. Dependence of relative variance of the laser beam intensity on the focusing parameter L/F in the turbulent atmosphere under conditions of precipitation: 1) typical dependence $\sigma_I^2(L/F)$ for comparable contribution from the components $\sigma^2_{I,T}$ and $\sigma^2_{I,P}$ to the measured variance; 2) component of $\sigma_I^2$ determined by the influence of precipitation, $\sigma^2_{I,P}$; 3) turbulent component of $\sigma_I^2$, $s^2_{I,T}$.*

## INTENSITY FLUCTUATIONS OF PARTIALLY COHERENT RADIATION

When analyzing turbulent fluctuations of radiation partially coherent in space and time it is necessary to take into account the fact that all the photodetectors responding to the intensity $I(x, \mathbf{R}, t) = |U(x, \mathbf{R}, t)|^2$ possess the finite response time and measure really the quantity averaged over a certain period $\tau_r$:

$$\overline{I(x, \mathbf{R}, t)} = \frac{1}{\tau_r} \int\limits_0^{\tau_r} I(x, \mathbf{R}, t + \tau)\, d\tau . \quad (14)$$

The complex amplitude of random field $U(x, \mathbf{R}, t)$ is characterized by the temporal scales $T_a$ and $\tau_s$. The first of them is the typical time of change of the atmospheric state, the second is determined by the coherence time of a source. Therefore, when investigating the intensity fluctuations of partially coherent radiation, it is necessary to take into account the relations between the characteristic times $T_a$, $\tau_s$ and $\tau_r$ (see Refs. 2, 53, and 54). Taking into consideration that the scale $T_a$ of change of atmospheric processes far exceeds, as a rule, the coherence time of a source and the response time of a receiver, let us deal with the two limiting situations $\tau_s \ll \tau_r$ and $\tau_s \geq \tau_r$.

### Averaging of fluctuations of the initial field by a receiver ($\tau_s \ll \tau_r$)

In this case the fourth moment of initial field $\langle U_0(t_1)\, U_0^*(t_2)\, U_0^*(t_3)\, U_0^*(t_4)\rangle$ needed for calculation of the turbulent intensity fluctuations of partially coherent laser beams is given as[2,55]:

$$\langle U_0(t_1)\, U_0^*(t_2)\, U_0^*(t_3)\, U_0^*(t_4)\rangle =$$
$$= \langle U_0(t_1)\, U_0^*(t_2)\rangle \langle U_0(t_3)\, U_0^*(t_4)\rangle . \quad (15)$$

It follows from an analysis of the obtained expressions for the relative variance of the intensity[53] that in this case the decrease of the degree of the source spatial coherence leads to lower light intensity fluctuations in a turbulent medium. In particular, the level of saturation of variance of the coherent beam intensity appears to be less than unity. It is determined by the ratio of the radius of the source spatial coherence $a_c$ to the initial radius of the beam itself[53]:

$$\sigma_I^2 = (1 + a^2 / a_c^2)^{-1} \leq 1. \quad (16)$$

Figure 12 shows the results of calculation of the dependence of the standard deviation of intensity $\sigma_I$ on the parameter $\Omega_c = k\, a_c^2 / L$, characterizing the degree of the source spatial coherence for narrow laser beam ($\Omega = 1$) and for a plane wave ($\Omega = 10^4$). The calculations were carried out for different values of the parameter $\beta_0^2$ corresponding to the regimes of weak and strong intensity fluctuations. The figure shows that with the decrease of the degree of spatial coherence of the initial wave field (parameter $\Omega_c$) the relative variance is decreased down to a certain level, not exceeding unity. The magnitude of this level varied depending on the Fresnel number of the source aperture $\Omega$ and on the parameter of turbulence $\beta_0^2$.

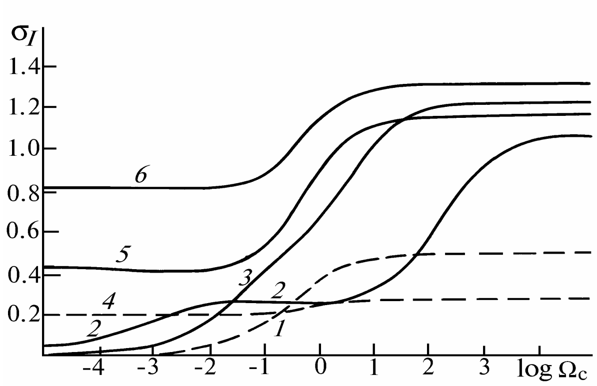



*FIG. 12. Dependence of standard deviation of the intensity $\sigma_I$ on the degree of radiation coherence for different values of the turbulence parameter $\beta_0$: 1–3) plane wave ($\Omega = 10^4$), $\beta_0 = 0.5$ (1), 1 (2), and 10 (3); 4–6) collimated beam ($\Omega = 1$), $\beta_0 = 0.5$ (4), 10 (5), and 2 (6).*

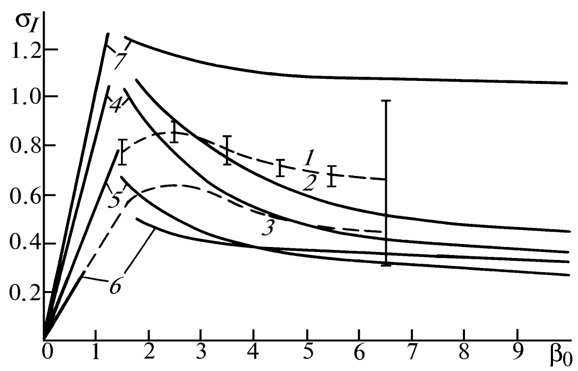



*FIG. 13. Dependence of standard deviation of the intensity fluctuations of partially coherent radiation on the parameter $\beta_0$: 1) experimental data from Ref. 56, $\Omega = 90$; 3) experimental data from Ref. 57, $\Omega = 21.5$ and 30; 2) $\Omega = 90$, $a^2/a_c^2 = 9$; 4) $\Omega = 90$, $a^2/a_c^2 = 20$; 5) $\Omega = 90$, $a^2/a_c^2 = 200$; 6) $\Omega = 21.5$ for $\beta_0 > 1$ and $\Omega = 30$ for $\beta_0 < 1$, $a^2/a_c^2 = 200$; and, 7) $\Omega = 90$, $a^2/a_c^2 = 0$.*

The measurements of the relative variance of the intensity of multimode radiation in the turbulent atmosphere were performed in Refs. 56 and 57. The obtained experimental data on the dependence of the relative variance $\sigma_I^2$ on the strength of optical turbulence are shown in Fig. 13 along with the results of calculations done in Ref. 53 for the parameter $\Omega$ corresponding to that of the experiments performed in Refs. 56 and 57. The lack of substantiated data on the laser radiation coherence in the above–indicated papers did not allow the quantitative comparison between theoretical and experimental results to be made; however, qualitatively these results are in good agreement.

Note that the theoretical model of turbulent fluctuations of partially coherent radiation for $\tau_s \ll \tau_r$ gives the results for the relative variance of the intensity that are qualitatively close to those observed in the experiments on measurement of the intensity fluctuations of laser radiation in precipitation (see the previous section) if we conventionally accept that screening of the output aperture of a source by precipitation particles at moderate optical depths is equivalent to partition of the intensity distribution of the initial field into "zones" of coherence (i.e., formation of the source with partial spatial coherence), and the increase of optical depth (intensity of precipitation) is equivalent to vanishing of spatial coherence of initial radiation.

The calculations of spatial intensity correlation of partially coherent radiation for $\tau_s \ll \tau_r$ (see Ref. 53) have shown that the degree of negative correlation typical of weak fluctuations in narrow beams (see Fig. 3) decreases with degradation of the source coherence, and the region of positive correlation extends. In the regime of strong fluctuations with lower degree of the source coherence residual correlation increases (see Fig. 3), and in the limiting case of incoherent source "the second" scale $L/k\rho_c$, characterizing the region of residual correlation in the case of coherent radiation (see section 3) becomes the sole scale of spatial correlation of the intensity.

The experimental study of spatial correlation of incoherent source intensity under conditions of strong fluctuations was carried out in Ref. 58. Figure 14 shows average experimental curves of the correlation coefficient obtained in that paper. This figure also shows the calculated dependences for the conditions of the experiment. As seen from the figure, the experimental and theoretical results are in good agreement.

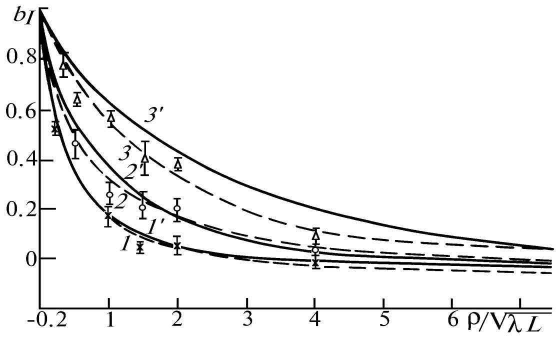



*FIG. 14. Intensity correlation coefficient of an incoherent light beam. Curves 1–3 correspond to experimental data from Ref. 58,* $\beta_0 = 1–3$ (*1*), 3–7 (*2*), *and* 7–11 (*3*). *Curves 1'– 3' correspond to calculated results from Ref. 53,* $\beta_0 = 2.7$ (*1'*), 4 (*2'*), *and* 7 (*3'*).

### Case of resolution of the initial field fluctuations by a receiver ($\tau_s \geq \tau_r$)

If the source is nearly incoherent so that the radius of spatial coherence of initial field satisfies the inequality $a_c < \sqrt{\lambda L}$ for the regime of weak optical turbulence, or the inequality $a_c < \rho_c$ for strong fluctuations, in the case $\tau_s \geq \tau_r$ the fourth moment of the initial field can be represented in the following form[54,55]:

$$<U_0(\mathbf{t}_1)\, U_0^*(\mathbf{t}_2)\, U_0(\mathbf{t}_3)\, U_0^*(\mathbf{t}_4)> =$$

$$= <U_0(\mathbf{t}_1)\, U_0^*(\mathbf{t}_2)><U_0(\mathbf{t}_3)\, U_0^*(\mathbf{t}_4)> +$$

$$+ <U_0(\mathbf{t}_1)\, U_0^*(\mathbf{t}_4)><U_0^*(\mathbf{t}_2)\, U_0(\mathbf{t}_3)> . \quad (17)$$

The use of boundary condition (17) enables us to reveal that in the case $\tau_s \geq \tau_r$ under study the relative variance of the axial intensity is related with that for $\tau_s \ll \tau_r$, $\sigma^2_{I\,S}$, by a simple relationship

$$\sigma_I^2 = 1 + 2\sigma^2_{I\,S} . \quad (18)$$

Thus due to the fact that a receiver responces to random source field intensity, the relative variance exceeds unity under any strength of optical turbulence in the atmosphere.

The results of calculations of the correlation coefficient for $\tau_s \geq \tau_r$ revealed that the structure of the spatial correlation of the intensity fluctuations in narrow laser beams remains qualitatively unchanged as for completely coherent sources. For sources emitting a plane wave ($\Omega \gg \max\{1,\ \beta_0^{12/5}\}$), the decrease of the degree of initial field coherence results in a noticeable decrease of the radius of spatial intensity correlation. In the limiting case $\Omega_c \to 0$, $\Omega \to \infty$ the intensity fluctuations for $\tau_s \geq \tau_r$ become delta–correlated.

In conclusion, it should be noted that this paper presents not all the results obtained at the Institute of Atmospheric Optics concerning the investigations of laser radiation intensity fluctuations in the turbulent atmosphere, which were mainly integrated experimental and theoretical in character. Because of limitations on the length of the paper, it appeared to be impossible to include in the review article the results of theoretical studies of the intensity fluctuations in laser beams under thermal blooming,[59,60] conducted at the Institute, to consider the potentialities of numerical simulation of radiation propagation in random media, and so on. These results along with the others can be found in Refs. 2 and 61 and in the original papers of scientists of our Institute.

### ACKNOWLEDGMENTS

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