Rate averaging in free space optics systems using incoherent sources

Yahya Baykal

Çankaya University, Electronic and Communication Engineering Department, Ankara, Turkey

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Effect of the information rate on the scintillation index is examined for free space optical (FSO) broadband access applications that use spatially incoherent sources. For this purpose, intensity fluctuations are formulated indicating the effect of the rate on the scintillation index in the presence of the atmospheric turbulence. The bandwidth of modulation of the incoherent source is taken to be much smaller than the carrier frequency, i.e., narrowband approximation is employed. Rate averaging factor for spatially incoherent source is derived as to represent the averaging in weak atmospheric turbulence due to rate of modulation of the intensity. It is found that the scintillations decrease as the rate of transmission through atmospheric turbulence increases. This decrease is independent of the carrier wavelength of the FSO system but depends on the outer scale of turbulence. Up to 10 Gb/s, the decrease is negligible for realistic outer scale values. When extremely large eddies are present in the formation of turbulence, rate can be effective in the reduction of the scintillations even at rates up to 10 Gb/s. In the limit when the information rate is taken as zero, our results correctly reduce to the known scintillations for spatially incoherent monochromatic excitation.

Introduction

Free Space Optics (FSO) communication systems have become one of the most widely used access systems especially in metropolitan applications. Current practical broadband access FSO systems that employ coherent laser sources are being used at bit rates up to 10 Gb/s mainly in horizontal links of a few kilometers length. FSO systems using incoherent LED sources are also practiced mainly due to their economical benefits and wider range of coverage. These systems operate usually at infrared carrier wavelengths of 0.85 and 1.55 μm.

Atmospheric turbulence induces considerable limitations in FSO access links, as it causes signal dependent noise in the form of intensity scintillations. Intensity fluctuations in FSO links and their reduction by various means are studied.[1–5] In the efforts of reducing the scintillations, averaging due to receiver and transmitter aperture sizes and due to the frequency content of the source are of interest.[6–10] Scintillations of ultrashort optical pulses in the turbulent medium is also reported.[11]

In this paper, we study the effect of the information rate on the scintillations when the FSO system uses a spatially incoherent source. We base our formulations on temporally coherent (i.e, monochromatic) but spatially incoherent sources that are modulated at rates much smaller than the carrier frequency, i.e., narrowband approximation is utilized. Narrowband approximation is realistic since even within such limitation, our analysis still covers the extremely high bit rates in the infrared which are currently far from being practically implemented. In our work, point to point link with a single carrier wavelength is examined and point detector is assumed. Also, Huygens–Fresnel principle is used for a spatially incoherent source which is intensity-modulated, the intensity having a Gaussian pulse shape.

We derived a practical formula showing the rate averaging effect to understand quantitatively how the scintillation levels decrease beyond their zero-information-bit-rate values and fall in the tolerable or improved scintillation index range when the information bit rate is increased. Rate averaging factor examined in our paper is a normalized quantity so it represents the improvement over the absolute scintillation level of the link. Thus, appreciably low rate averaging factor means that the performance of an atmospheric optical link is improved (considering turbulence only) when it is operated at a higher information rate which can be a factor in the design of the link.

Our results show an averaging effect of the information rate on the scintillations, i.e., as the information rate increases, the scintillations are smoothed out. For a spatially incoherent finite sized source in weak turbulence, rate averaging effect is found to be dependent on the outer scale of turbulence. Within the currently practical information rates that are possible to implement, i.e., for rates up to 2.5 Gb/s or 10 Gb/s, it is found that the rate averaging factor is negligible for realistic outer scale values which are less than 100 meters. However, when extremely large sizes of eddies are present in the formation of turbulence then rate averaging can also be effective even at rates up to 10 Gb/s.

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2. Rate averaging for spatially incoherent source in weak turbulence

In our recent work[10] we have formulated the scintillation index in turbulence for coherent sources having a finite information rate as the result of intensity modulation. Here, we start by outlining the initial steps of the same formulation, then introduce a spatially incoherent finite size beam wave source with size α_s, and find the effect of the information rate on the scintillation index.

One can find the instantaneous intensity as

$$I = (1/T_d)\int_{T_0-T_d/2}^{T_0+T_d/2} dt\left|u(L,\ \mathrm{p},\ t)\right|^2, \quad (1)$$

where T_0 is a reference time, T_d is the response time of the detector, p is the transverse coordinates at the receiver plane, $u(L, \mathrm{p}, t)$ is the field at $z = L$, i.e., at the receiver plane, L being the path length. Time variation of the field at the receiver is given by the Fourier transform relation as

$$u(L,\ \mathrm{p},\ t) = 2\pi\int_{-\infty}^{\infty} df\ u(L,\ \mathrm{p},\ f)\exp(i2\pi ft), \quad (2)$$

$u(L, \mathrm{p}, f)$ is the solution by the extended Huygens–Fresnel principle and f is the frequency. For a polychromatic radiation where the bandwidth is centered about the carrier frequency f_0

$$u(L,\ \mathrm{p},\ t) = \exp[i2\pi f_0(t + L/c)]\times$$
$$\times\int_{-\infty}^{\infty} 2\pi\ df_1 A_p(L,\ \mathrm{p},\ f_0+f_1)\exp[i2\pi f_1(t + L/c)] +$$
$$+\exp[-i2\pi f_0(t + L/c)]\times$$
$$\times\int_{-\infty}^{\infty} 2\pi\ df_2 A_n(L,\ \mathrm{p},\ f_2-f_0)\exp[i2\pi f_2(t + L/c)], \quad (3)$$

where A_p and A_n are the positive and negative portions of the complex temporal frequency spectrum at the receiver plane, respectively. A_p is given by

$$A_p(L,\ \mathrm{p},\ f_0+f_1) = (f_0+f_1)/(icL)\times$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathrm{s}\ A_{ps}(f_0+f_1)u(0,\ \mathrm{s},\ f_0+f_1)\times$$
$$\times\exp[i\pi(c/L)(f_0+f_1)(\mathrm{p}-\mathrm{s})^2 + \psi(\mathrm{s},\ \mathrm{p},\ f_0+f_1)]. \quad (4)$$

Similarly, one can obtain A_n by inserting $f_2 - f_0$ in place of $f_0 + f_1$ in Eq. (4). Here A_{ps} and A_{ns} are the positive and negative portions of the complex temporal frequency spectrum of the source, $u(0, \mathrm{s}, f_0 + f_1)$ represents the field at the source plane at frequency $f_0 + f_1$, s is the transverse source coordinate, L is the path length, c is the speed of light in free space, $\psi(\mathrm{s}, \mathrm{p}, f_0 + f_1)$ is the solution to Rytov method representing the random part of the complex phase of a spherical wave at frequency $f_0 + f_1$ propagating from source point (0, s) to observation point (L, p).

In writing A_p and A_n, temporal frequency spectrum of the source due to digital intensity modulation and the spatial field distribution of the source are assumed to be separable. Substituting A_p in Eq. (4) and A_n into Eq. (3) and using the hermitian property for the source spectrum (i.e., $A^*_{ps}(f) = A_{ns}(f)$ and $A^*_{ps}(-f) = A^*_{ns}(f)$, where "*" denotes the complex conjugation), the instantaneous intensity for a point receiver at the optical axis p = 0 is found as:

$$I = 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(2\pi)^2 df_1 df_2 (f_0+f_1)(f_0+f_2)/(cL)^2 P_c(f_0+f_1, f_0+f_2)\times$$
$$\times\mathrm{sinc}[\pi T_d(f_1-f_2)]\exp[i2\pi(f_1-f_2)(T_0+L/c)]\times$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathrm{s}_1\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathrm{s}_2\,\Gamma^s_2(\mathrm{s}_1,\ \mathrm{s}_2,\ f_0+f_1,\ f_0+f_2)\times$$
$$\times\exp\{i\pi/(Lc)[(f_0+f_1)\mathrm{s}_1^2-(f_0+f_2)\mathrm{s}_2^2]\}\times$$
$$\times\exp[\psi(\mathrm{s}_1,\mathbf{0},f_0+f_1)+\psi^*(\mathrm{s}_2,\mathbf{0},f_0+f_2)], \quad (5)$$

where

$$P_c(f_0+f_1, f_0+f_2) = \langle A_{ps}(f_0+f_1)A^*_{ps}(f_0+f_2)\rangle_s$$

in general represents the cross frequency spectrum of the intensity of the pulse which contains both the source temporal coherence centered around the carrier frequency f_0 and the intensity modulation. $\Gamma^s_2(\mathrm{s}_1, \mathrm{s}_2, f_0 + f_1, f_0 + f_2)$ is the second order source mutual coherence function including spatial coherence effects and is defined as

$$\Gamma^s_2(\mathrm{s}_1,\ \mathrm{s}_2,\ f_0+f_1,\ f_0+f_2) =$$
$$= \langle u(0,\ \mathrm{s}_1,\ f_0+f_1)u^*(0,\ \mathrm{s}_2,\ f_0+f_2)\rangle_s =$$
$$= \langle u_d(0,\ \mathrm{s}_1,\ f_0+f_1)u^*_d(0,\ \mathrm{s}_2, f_0+f_2)u_r(0,\ \mathrm{s}_1,\ f_0+f_1)\times$$
$$\times u^*_r(0,\ \mathrm{s}_2,\ f_0+f_2)\rangle_s. \quad (6)$$

Here $u_d(0, \mathrm{s}_1, f_0 + f_1)$ and $u_r(0, \mathrm{s}_1, f_0 + f_1)$ represent the deterministic part of the source field and the random part of the field due to spatial decorrelation, respectively.

Note that Eq. (6) is written under the condition that many samples of the random part of the incident field are present within the response time of the detector, T_d. Regarding the turbulence part, coherence time of the medium (≈ 1 ms) is much longer than T_d.

In this paper we examine the averaging of the rate of modulation on the scintillation index thus the effect of source temporal coherence is discarded, i.e., the light source to be modulated is assumed to be monochromatic. Thus

$$P_c(f_0+f_1, f_0+f_2) = A_{ps}(f_0+f_1)A^*_{ps}(f_0+f_2)$$

represents the cross frequency spectrum of the intensity of the pulse which contains the intensity modulated information centered around the carrier frequency f_0. In practice, unequal frequency components will not contribute much to this frequency spectrum. Most contribution will come from the close frequency components so we approximated the cross frequency spectrum of the intensity of the pulse $P_c(f_0 + f_1, f_0 + f_2)$ by a delta function representation as

$$P_c(f_0 + f_1, f_0 + f_2) = P(f_0 + f_1)\delta(f_1 - f_2), \quad (7)$$

where $\delta(f_1 - f_2)$ is the Dirac delta function, $P(f_0 + f_1)$ is the frequency spectrum of the intensity of the pulse which contains the intensity modulated information. In other words $P(f_0 + f_1)$ is the Fourier transform of the time waveform of the intensity pulse. Here we note that using delta approximation in Eq. (7) substantially easies the formulation, however brings certain inaccuracy in the results, closer to being exact only for very wide pulses. Thus, our results are assumed to be acceptable in the narrowband approximation, more accurate for slower pulse rates.

Using Eq. (7) in Eq. (5) and carrying out the integration over f_2, the instantaneous intensity becomes

$$I = 8\pi^2 / (cL)^2 \int_{-\infty}^{\infty} df_1 (f_0 + f_1)^2 P(f_0 + f_1) \times$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2 s_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2 s_2 \; \Gamma_2^s(s_1, s_2, f_0 + f_1, f_0 + f_1) \times$$
$$\times \exp\{i\pi / (Lc)[(f_0 + f_1)(s_1^2 - s_2^2)]\} \times$$
$$\times \exp[\psi(s_1, \mathbf{0}, f_0 + f_1) + \psi^*(s_2, \mathbf{0}, f_0 + f_1)]. \quad (8)$$

For spatially incoherent source, the second order source mutual coherence function Γ_2^s is given by δ-function [Eqs. (20)–(74) of Ref. 12] and for $k = 2\pi(f_0 + f_1)/c$, Γ_2^s is given by

$$\Gamma_2^s(s_1, s_2, f_0 + f_1, f_0 + f_1) =$$
$$= [c / (f_0 + f_1)]^2 I[(s_1 + s_2) / 2]\delta(s_1 - s_2), \quad (9)$$

where $I[(s_1 + s_2)/2]$ is the intensity at $(s_1 + s_2)/2$ on the source plane. Substituting Eq. (9) into Eq. (8),

$$I = 8\pi^2 / L^2 \int_{-\infty}^{\infty} df_1 P(f_0 + f_1) \times$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2 s_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2 s_2 I[(s_1 + s_2) / 2]\delta(s_1 - s_2) \times$$
$$\times \exp\{i\pi / (Lc)[(f_0 + f_1)(s_1^2 - s_2^2)]\} \times$$
$$\times \exp[\psi(s_1, \mathbf{0}, f_0 + f_1) + \psi^*(s_2, \mathbf{0}, f_0 + f_1)], \quad (10)$$

where

$$\psi(s_1, 0, f_0 + f_1) = \chi(s_1, 0, f_0 + f_1) + iS(s_1, 0, f_0 + f_1);$$

ψ, χ, and S are the wave, log-amplitude and phase fluctuations at frequency $f_0 + f_1$, respectively. Performing the integration over s_2, Eq. (10) reduces to

$$I = 8\pi^2 / L^2 \int_{-\infty}^{\infty} df_1 P(f_0 + f_1) \times$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2 s_1 I(s_1) \exp[2\chi(s_1, 0, f_0 + f_1)]. \quad (11)$$

Using the conservation of energy in weak turbulence, i.e., $\langle \exp[2\chi(s_1, 0, f_0 + f_1)] \rangle_m = 1$, the average intensity for the spatially incoherent pulse is found from Eq. (11) as

$$\langle I \rangle = 8\pi^2 / L^2 \int_{-\infty}^{\infty} df_1 P(f_0 + f_1) \int_{-\infty}^{\infty} d^2 s_1 I(s_1). \quad (12)$$

For the intensity distribution at the source plane, Gaussian profile is taken as

$$I(s_1) = \exp(-s_1^2 / \alpha_s^2), \quad (13)$$

where α_s is the source size. For the time waveform of the intensity pulse, Gaussian shaped pulse is chosen as

$$p(t) = [1 / (4\pi)] \exp(-t^2 / T_b^2), \quad (14)$$

where T_b denotes the effective duration of one bit. Taking the Fourier transform of the Gaussian pulse and shifting around the carrier frequency f_0 yields

$$P(f) = [1 / (2\pi^{1/2} R_b)] \exp\{-[2\pi(f_0 - f) / R_b]^2\}, \quad (15)$$

where $R_b = 2/T_b$ is defined as the information rate to be transmitted in the free space optics system.

Substituting Eqs. (13) and (15) into Eq. (12), performing the integrations by using Eq. (3.321.3) of Ref. 14, the average intensity for the spatially incoherent pulse is found as

$$\langle I \rangle = 2(\pi\alpha_s / L)^2. \quad (16)$$

In finding $\langle I^2 \rangle$, Eq. (11) is used to have

$$\langle I^2 \rangle = 8\pi^2 / L^2 \int_{-\infty}^{\infty} df_1 P(f_0 + f_1) \times$$
$$\times \int_{-\infty}^{\infty} df_2 P(f_0 + f_2) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2 s_1 I(s_1) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2 s_2 I(s_2) \times$$
$$\times \langle \exp[2\chi(s_1, 0, f_0 + f_1) + 2\chi(s_2, 0, f_0 + f_2)] \rangle_m. \quad (17)$$

For χ Gaussian distributed and in weak turbulence, $\langle\ \rangle_m$ in Eq. (17) can be approximated as

$$\langle \exp[2\chi(s_1, 0, f_0 + f_1) + 2\chi(s_2, 0, f_0 + f_2)] \rangle_m \approx$$
$$\approx 1 + 4B_\chi(s_1, s_2, f_0 + f_1, f_0 + f_2), \quad (18)$$

where $B_\chi(s_1, s_2, f_0 + f_1, f_0 + f_2)$ is the two source, two frequency spherical wave covariance function of the log-amplitude fluctuations defined as

$$B_\chi(\mathbf{s}_1,\mathbf{s}_2,f_0+f_1,f_0+f_2)=\langle[\chi(\mathbf{s}_1,0,f_0+f_1)-$$

$$-\langle\chi(\mathbf{s}_1,0,f_0+f_1)\rangle_m]\,[\chi(\mathbf{s}_2,0,f_0+f_2)-\langle\chi(\mathbf{s}_2,0,f_0+f_2)\rangle_m]\,\rangle_m. \quad (19)$$

For B_χ, we have used the approximation given by Eq. (12) of Fante's paper,[15] i.e.,

$$B_\chi(\mathbf{s}_1,\mathbf{s}_2,f_0+f_1,f_0+f_2)\approx$$

$$\approx\sigma_\chi^2\exp[-|\mathbf{s}_1-\mathbf{s}_2|^2\rho_0^{-2}-(2\pi)^2(f_1-f_2)^2\Omega^{-2}], \quad (20)$$

where $\sigma_\chi^2=0.124k_0^{7/6}C_n^2L^{11/6}$ is the variance of the log amplitude fluctuations for a spherical wave operating at the carrier frequency f_0 (note that the coefficient of σ_χ^2 in Ref. 15 should be corrected as 0.124), $\rho_0=(0.545k_0^2C_n^2L)^{-3/5}$ is the coherence length of the medium, $k_0=2\pi f_0/c$ is the wave number at the carrier frequency, $\Omega^{-2}=0.39C_n^2L_0^{5/3}Lc^{-2}$, C_n^2 is the structure constant, L_0 is the outer scale of turbulence.

Here we note that in our representation of the covariance [given by Eq. (20)] for the spatially incoherent pulse, we followed the same representation used by Fante[15] since in his paper he also studied a spatially incoherent source with finite spectral extent (due to temporal source coherence). The detailed derivation of Ω^{-2}, i.e., the outer scale dependence of the covariance function of the log-amplitude fluctuations due to the finite spectral content of the source is presented in Fante's earlier paper[16] [Eq. (12) in Ref. 16].

Substituting Eqs. (13), (15), (18) and (20) into Eq. (17), $\langle I^2\rangle$ for the spatially incoherent pulse is obtained as

$$\langle I^2\rangle=8\pi^2/L^2\int_{-\infty}^{\infty}\mathrm{d}f_1[1/(2\pi^{1/2}R_\mathrm{b})]\exp[-(2\pi)^2f_1^2/R_\mathrm{b}^2]\times$$

$$\times\int_{-\infty}^{\infty}\mathrm{d}f_2[1/(2\pi^{1/2}R_\mathrm{b})]\exp[-(2\pi)^2f_2^2/R_\mathrm{b}^2]\times$$

$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}^2\mathbf{s}_1\exp(-\mathbf{s}_1^2/\alpha_\mathrm{s}^2)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}^2\mathbf{s}_2\exp(-\mathbf{s}_2^2/\alpha_\mathrm{s}^2)\times$$

$$\times\{1+4\sigma_\chi^2\exp[-|\mathbf{s}_1-\mathbf{s}_2|^2\rho_0^{-2}-(2\pi)^2(f_1-f_2)^2\Omega^{-2}]\}. \quad (21)$$

Making the transformations $f_\mathrm{c}=(f_1+f_2)/2$, $f_\mathrm{d}=f_1-f_2$, $\mathbf{s}_\mathrm{c}=(\mathbf{s}_1+\mathbf{s}_2)/2$, $\mathbf{s}_\mathrm{d}=\mathbf{s}_1+\mathbf{s}_2$, in Eq. (21), rearranging, and performing the integrations in Eq. (21) by using Eq. (3.321.8) of Ref. 14, $\langle I^2\rangle$ reduces to

$$\langle I^2\rangle=[2(\pi\alpha_\mathrm{s}/L)^2]^2(1+4\sigma_\chi^2/\{[1+(2\alpha_\mathrm{s}^2/\rho_0^2)]\times$$

$$\times[1+(T^2R_\mathrm{b}^2)]^{1/2}\}). \quad (22)$$

Here R_b is the rate in Gb/s, $T=(8.667C_n^2L_0^{5/3}L)^{1/2}$ is defined as the turbulence factor of the rate averaging for spatially incoherent source.

Using equations (16) and (22), the scintillation index

$$m^2=\left(<I^2>-<I>^2\right)/<I>^2 \quad (23)$$

is found as

$$m^2=4\sigma_\chi^2/\{[1+(2\alpha_\mathrm{s}^2/\rho_0^2)][1+(T^2R_\mathrm{b}^2)]^{1/2}\}. \quad (24)$$

The scintillation index presented by Eq. (24) refers to spatially incoherent finite-sized pulse source in weak turbulence and it includes the turbulence outer scale. As the limiting case of our result in Eq. (24) for no modulation, i.e., $R_\mathrm{b}=0$, the scintillation index becomes $4\sigma_\chi^2/[1+(2\alpha_\mathrm{s}^2/\rho_0^2)]$, which is the scintillation index due to a spatially incoherent monochromatic finite-sized source in weak turbulence. This is the same result obtained in the spatially incoherent monochromatic beam wave limit of a spatially and temporally partially coherent beam wave solution in weak turbulence.[17] Thus the term

$$\mathrm{RAF_{sis}}=1/\left[1+\left(T^2R_\mathrm{b}^2\right)\right]^{1/2} \quad (25)$$

in Eq. (24) which we name as $\mathrm{RAF_{sis}}$ (Rate Averaging Factor) for spatially incoherent source is identified as to represent the averaging in weak atmospheric turbulence due to rate of modulation of the intensity. As seen from the rate averaging factor, the scintillations decrease as the rate of transmission through atmospheric turbulence increases.

2. Results and conclusions

Rate averaging factors examined in this paper are normalized quantities so they present improvements over the absolute scintillation levels of the link. Thus, appreciably low rate averaging factor means that the performance of an atmospheric optical link is improved (considering turbulence only) when it is operated at a higher information rate, which can be a factor in the design of the link.

Equation (25) shows that in weak atmospheric turbulence, the rate averaging factor for spatially incoherent source ($\mathrm{RAF_{sis}}$) is dependent on the path length (L), outer scale of turbulence (L_0), and the structure constant, however it is independent of the carrier wavelength of the free space communication system. Equation (25) is plotted in Fig. 1 for various outer scale of turbulence, L_0.

For smaller L_0, rate averaging is ineffective even for very large rates. For very large outer scale values when spatial frequencies close to zero exist in the power spectrum of the refractive index, i.e., when the turbulence can be presented by Kolmogorov spectrum, then there can be appreciable reduction of scintillations especially at very high rates.

Figure 2 is plotted to better understand the rate averaging effect for spatially incoherent source within the currently practical information rates that are possible to implement, i.e., for R_b up to 2.5 Gb/s or 10 Gb/s. It is observed in Fig. 2 that rate averaging factor is negligible for realistic outer scale values, which are less than 100 meters. However when extremely large sizes of eddies are present in the formation of turbulence then rate averaging can also be effective even at rates up to 10 Gb/s. In an atmospheric link, the outer scale of turbulence, L_0 can vary practically between 1 m to 100 m.

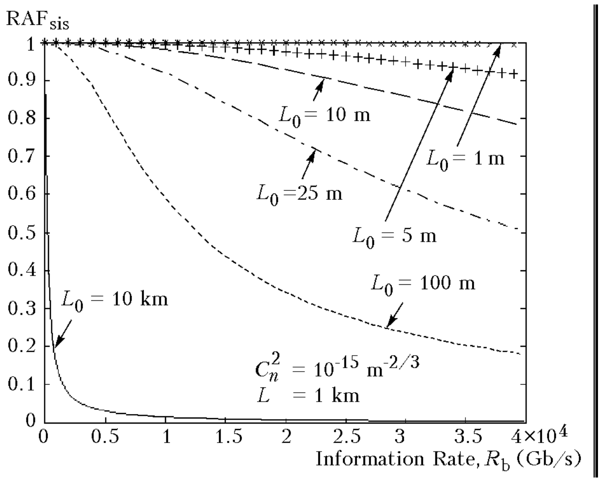

Fig. 1. Rate Averaging Factor for spatially incoherent source versus rate in weak turbulence for various outer scales of turbulence, L_0.

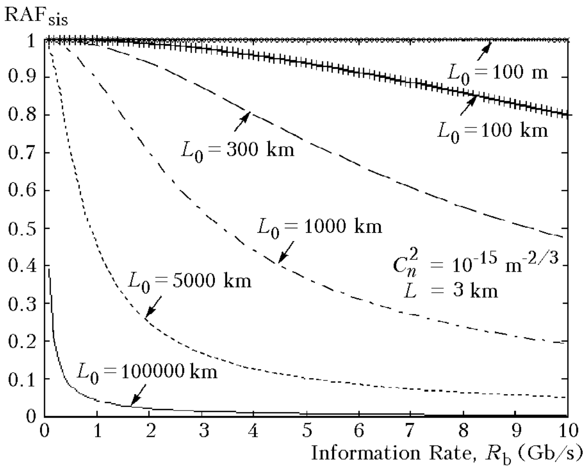

Fig. 2. Rate Averaging Factor for spatially incoherent source versus rate in weak turbulence for various L_0 within the range of currently practical information rates.

In examining Figs. 1 and 2, the worst-case value (i.e., the maximum value) of the rate averaging factor should be considered in one specific link. For example in Fig. 2, if the turbulence in the link operating at 10 Gb/s has outer scale changing between 100 m to 100 km, then the effective RAF$_{sis}$ should be 1 (i.e., the worst case) but not 0.8. Also we note that the instances when bandwidth effects are significant in Figs. 1 and 2 cover the extreme ranges, i.e., either when the bit rates are extremely high and/or when the outer scales of turbulence are extremely large.

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