To sodar measurements of parameters of the module and direction of the horizontal wind velocity

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Algorithms for determination of the means, standard deviations, asymmetry coefficients, and excess of the horizontal wind speed and direction from Volna-3 sodar measurements are described. Two possible approaches to the construction of the algorithms and the estimation of their errors are considered. Some vertical profiles of the characteristics of interest along with the confidence intervals are exemplified as determined from the sodar measurements.

Introduction

The parameters of the longitudinal u and cross v components of the horizontal wind velocity vector V_h were estimated in Refs. 1 and 2 by means of sodars. These characteristics often serve as a basis for indirect determination of the parameters of the module V_m and the direction φ of this vector. Then, the standard deviation $\sigma(\varphi)$ is estimated[2] based on the relation

$$\theta'(i) = v(i) / M(u), \tag{1}$$

where $\theta'(i)$ is the deviation of the current values $\varphi(i)$ from the direction θ of the mean vector $M(V_h)$ (Ref. 1);

$$M(u) = \sqrt{M^2(V_x) + M^2(V_y)}$$

is the module $M(V_h)$ coinciding with the mean value of the u-component, $M(V_x)$, $M(V_y)$ are the means of the orthogonal components of the vector $M(V_h)$ in some Cartesian coordinate system. (Here and below all directions are treated in meteorological meaning). Formula (1) corresponds to the linear part of the expansion in Taylor series in the vicinity of the mean values $M(u)$ and $M(v) = 0$ of the initial relationship for θ' (Ref. 2)

$$\theta'(i) = \arctan[v(i) / u(i)], \quad -\pi \le \theta' < \pi. \tag{2}$$

The approximation (1) is assumed true if pulsations of the wind direction do not exceed 20–30°. Then $\sigma(\varphi)$ is determined by the intensity of turbulence[3–5] I_v for v-component

$$\sigma(\varphi) = \sigma(\theta') = I_v = \sigma(v) / M(u). \tag{3}$$

And it follows for the asymmetry γ and excess ε coefficients

$$\gamma(\varphi) = \gamma(\theta') = \gamma(v), \quad \varepsilon(\varphi) = \varepsilon(\theta') = \varepsilon(v). \tag{4}$$

Formula (1) is widely used in the study of the structure of the boundary layer of the atmosphere by means of high meteorological masts.[3,5]

The basis for indirect measurements of the V_m components is the relationship

$$V_m = u + v^2 / 2M(u). \tag{5}$$

It can be obtained through expanding the initial formula relating the current values of $V_m(i)$ with corresponding orthogonal components of the horizontal wind velocity V_h, into Taylor series up to square terms[6]

$$V_m(i) = \sqrt{V_x^2(i) + V_y^2(i)} = \sqrt{u^2(i) + v^2(i)}. \tag{6}$$

The relation between V_m mean values and the longitudinal component of the vector V_h follows from Eq. (5)

$$M(V_m) = M(u)(1 + I_v^2 / 2). \tag{7}$$

To determine the standard deviation $\sigma(V_m)$, we use the well-known statistical relationship $\sigma^2(\cdot) = A_2(\cdot) - M^2(\cdot)$, where A_2 is the second initial moment. It follows from Eq. (6) that

$$A_2(V_m) = M(V_m^2) = \sigma^2(u) + M^2(u) + \sigma^2(v).$$

Then, taking into account Eq. (7), we obtain

$$\sigma(V_m) = \sigma(u)\sqrt{1 - I_v^2\sigma^2(v) / 4\sigma^2(u)}. \tag{8}$$

From Eq. (5) we also can obtain the relationships connecting the asymmetry and excess coefficients of V_m with parameters of uv-components. These relationships include central moments $\mu(v)$ of the 6th and 8th orders. However, in practical realizations of the indirect method, replacement of $\mu_6(v)$ and $\mu_8(v)$ by their sampling values $\hat{\mu}_6(v)$, $\hat{\mu}_8(v)$ leads to large errors in estimating $\gamma(V_m)$ and $\varepsilon(V_m)$ because of low accuracy of their determination at a limited quantity of observations N. The reducing of these relationships to Gaussian case through replacement of $\mu_6(v)$ and $\mu_8(v)$ by the precisely measured $\mu_2(v)$, using the known functional dependences, is also unacceptable at a deviation of

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the v-component distribution from the normal one. Nevertheless, different approximate relations for $\gamma(V_m)$ with parameters of uv-components are presented in Ref. 3:

$$\gamma(V_m) = \gamma(u) + \sigma^2(v) / M(V_m)\sigma(u). \quad (9)$$

and in Ref. 6

$$\gamma(V_m) = \gamma(u) + 3\sigma^2(v) / M(u)\sigma(u). \quad (10)$$

One of the purposes of this paper is practical testing of the efficiency of the indirect estimates of parameters of the module V_m and the direction φ of the horizontal wind velocity vector based on the expansions (1) and (5) as applied to acoustic sensing of the atmosphere. To do this, we compare the results with analogous data obtained by so-called "direct" method,[1,5,6] which is not based on the expansions (1) and (5). Another purpose is to obtain the standard errors and the values of 90% confidence intervals for the parameters of V_m and φ directly from the experimental data and to construct the measurement algorithms for two aforementioned variants.

1. Analysis of estimations of parameters of the horizontal wind velocity module

First, consider the "direct" method for estimating parameters of V_m. In its realization, the current values $V_m(i)$ obtained by the relationships (6) are considered as the results of direct measurements. Therefore, further determination of parameters of V_m (mean values, standard deviation, asymmetry and excess coefficients) and their point and interval errors can be performed analogously to estimating the parameters of the radial components V_r of the total wind velocity vector V, which is described in detail in Ref. 7.

Advantages and disadvantages of this method for sodar measurements are described in Ref. 1. The greatest disadvantage is the impossibility of calculation of "instantaneous" values of the uv-components in some sensing cycles at disappearing of the return signal in at least one of the radial channels of the sodar, which results in worsening of the measurement accuracy (the increase of corresponding confidence intervals) and in decrease of the sensing height relative to the potentially possible height, especially at small times of averaging.

When realizing the indirect method, replacing in Eqs. (7) and (8) the true moments of uv-components of the vector V_h by their estimates, we obtain relationships for calculation of means and standard deviations of the horizontal wind velocity:

$$\hat{M}(V_m)_k = \hat{M}(u) + \hat{D}(v) / 2\hat{M}(u),$$
$$\hat{\sigma}(V_m)_k = \sqrt{\hat{D}(u) - \hat{D}^2(v) / 4\hat{M}^2(u)}, \quad (11)$$

where formulas for estimates of the mean values $\hat{M}(u)$ and the variances $\hat{D}(u)$, $\hat{D}(v)$ are given in Ref. 1. (Here and below the estimates of the parameters of V_m and φ, corresponding to the indirect method, are marked by the index k).

To determine the standard measurement errors in Eq. (11), we use the method of linearization, i.e., we consider only the linear terms in corresponding Taylor series. The accounting for the nonlinear terms is not expedient in this case from a practical point of view because of the necessity to use further sampling moments of high orders, which are estimated with large errors at a limited quantity of observations. Then, after necessary averaging, we obtain the sought relationships

$$\sigma[\hat{M}(V_m)_k] = \Big\{(1 - I_v^2/2)^2 D[\hat{M}(u)] + D[\hat{D}(v)] / 4M^2(u) + (1 - I_v^2/2)\mathrm{cov}[\hat{M}(u), \hat{D}(v)] / M(u)\Big\}^{1/2},$$

$$\sigma[\hat{\sigma}(V_m)_k] = \Big\{4D[\hat{D}(u)] + I_v^4 D[\hat{D}(v)] + I_v^6\sigma^2(v)D[\hat{M}(u)] - 4I_v^2\mathrm{cov}[\hat{D}(u), \hat{D}(v)] + 4I_v^3\sigma(v)\mathrm{cov}[\hat{M}(u), \hat{D}(u)] - 2I_v^5\sigma(v)\mathrm{cov}[\hat{M}(u), \hat{D}(v)]\Big\}^{1/2} / 4\sigma(V_m),$$

The formulas for the variances of the estimates of corresponding moments of uv-components and their covariances are presented in Refs. 1 and 4 or directly follow from them. It follows from the presented relationships that in the majority of practical situations the standard errors in measurements of $\hat{M}(V_m)_k$ and $\hat{\sigma}(V_m)_k$ are mainly determined by random errors (variances) of the estimates of the analogous values of the longitudinal component u of the vector V_h.

Experimental results

We present the examples of measuring the vertical profiles of the horizontal wind velocity with the Volna-3 sodar (IAO SB RAS) by two aforementioned methods (Figs. 1–4). The measurements were conducted in the suburb of Tomsk city in the evening or at night on November 20, 1999 at averaging time T_{av} = 60 min and correspond to the profiles of different parameters of uv-components of the vector V_h presented in Ref. 1 (Figs. 2–4) and in Ref. 4 (Figs. 1–3). Without going into detailed physical interpretation of the obtained data, compare the used estimating methods and show their actual accuracy characteristics reached at the given T_{av}. To do this, the values of the corresponding 90% confidence intervals $I_{0.9}$ are plotted in all graphs. Here and below, the values were determined according to Refs. 1, 4, and 7 and based on preliminary calculation of standard errors in measuring the considered parameters. For a greater obviousness, only one-side $I_{0.9}$ intervals are shown in some figures. The superlinear signs "^" (mark of the estimate) are omitted in all figures.

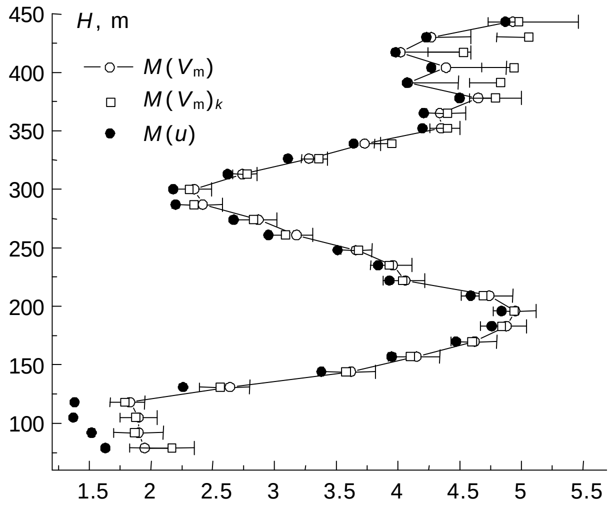

Fig. 1. Mean value of the module and u-component of the horizontal wind velocity, m/s.

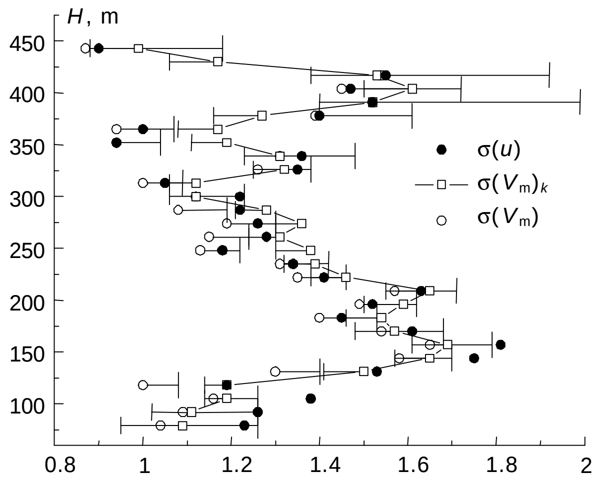

Fig. 2. Standard deviation of the module and u-component of the horizontal wind velocity, m/s.

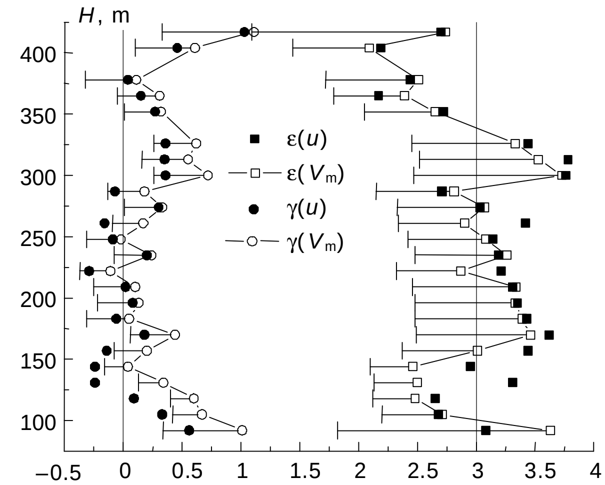

Fig. 3. Asymmetry and excess coefficients of the module and u-component of the vector V_h.

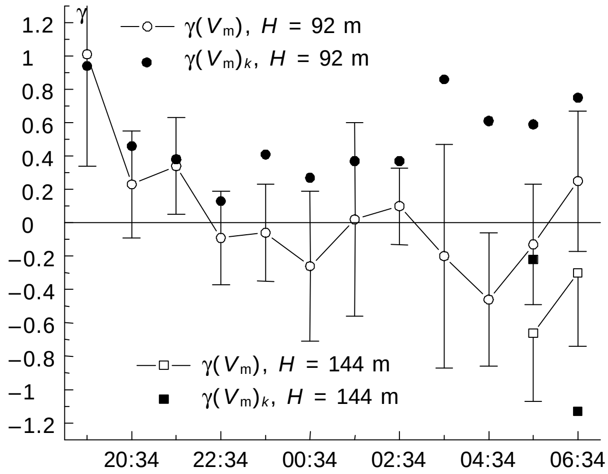

Fig. 4. Hourly variations of the asymmetry coefficient of the horizontal wind velocity.

Rather complicated behaviors of the vertical dependences of the V_m means obtained at 19:34 are shown in Fig. 1. They are similar to analogous profiles of the mean wind velocity $M(V_h)$ (i.e., actually $M(u)$ measured at the same time and presented in Ref. 1, Fig. 2). The deviation of $\hat{M}(V_m)$ from $\hat{M}(V_m)_k$ is insignificant in the greatest part of the height range at the presence of a stable return signal in all three radial channels of the sodar. The deviation increases only above ~ 380 m, and at two heights the confidence intervals of $\hat{M}(V_m)$ and $\hat{M}(V_m)_k$ even do not overlap. This can be explained by the fact that, as the return signal power decreases, $\hat{M}(V_m)$ can be determined only from some part of the measured radial components $V_r(i)$ of the vector V due to the aforementioned disadvantage of the "direct" method. But the estimate $\hat{M}(V_m)_k$ uses the whole obtained statistical ensemble of data. However, the maximal difference between $\hat{M}(V_m)$ and $\hat{M}(V_m)_k$ is small and does not exceed 0.8 m/s. In principle, when applying the indirect method, essential false increases of $\hat{M}(V_m)_k$ point values can be observed, which are attributed mainly to a small time of averaging T_{av}. Appearance of doubtfully high values of $\hat{D}(v)$ is possibly due to a small amount of data in the statistical ensembles $V_r(i)$ under processing,[1] which just leads to the increase of the results of calculation of $\hat{M}(V_m)_k$ [Eq. (11)]. But, as a rule, this effect is well manifests itself by a sharp increase of the confidence intervals of both $\hat{D}(v)$ and $\hat{M}(V_m)_k$, and therefore, it can be identified. The vertical profile of mean values of the longitudinal component $M(u)$ obtained by the "direct" method is also shown in Fig. 1. It is seen that the difference between $\hat{M}(V_m)$ and $\hat{M}(u)$ is maximal at small wind velocities, but in this case it is small and does not exceed 0.6 m/s. The greater differences, sometimes up to several meters per second, were observed in other situations.

Quite high correlation of point values of the standard deviations V_m obtained by two methods is seen in Fig. 2, as well as in Fig. 1. However, the number of significant differences between $\hat{\sigma}(V_\mathrm{m})$ and $\hat{\sigma}(V_\mathrm{m})_k$ is three times greater than in the previously considered case, and they are present even in the lower part of the height range. Nevertheless, the difference between $\hat{\sigma}(V_\mathrm{m})$ and $\hat{\sigma}(V_\mathrm{m})_k$ is small, it does not exceed 0.25 m/s. As for possible appearance of anomalous values $\hat{\sigma}(V_\mathrm{m})_k$ they, as before, are mainly accompanied by the enhanced $I_{0.9}$. The maximal differences between $\hat{\sigma}(u)$ and $\hat{\sigma}(V_m)$ are also observed at small wind velocities and make approximately 0.2 m/s.

On the whole, it is necessary to note, based on the practical application in the Volna-3 sodar of the indirect method to estimation of mean values and standard deviations of V_m, that it is expedient to check the results obtained at small values of the mean longitudinal component $M(u)$ (approximately up to 2–3 m/s) by analogous data of the "direct" method. It is explained by possible disability of the approximate formulas (7) and (8), because their application assumes insignificant pulsations of the uv-components relative to $M(u)$.[6]

Vertical profiles of the asymmetry and excess coefficients of the module and the longitudinal component of V_m obtained by the "direct" method are shown in Fig. 3. It is seen that the values of $\hat{\gamma}(V_\mathrm{m})$ are always greater than $\hat{\gamma}(u)$ at all heights. This was noted by many authors (see, for example, Refs. 3 and 6). Similar unambiguous agreement between the point values of $\hat{\varepsilon}(V_\mathrm{m})$ and $\hat{\varepsilon}(u)$ is not observed. At the same time, their measurement is accompanied by quite high values of $I_{0.9}$, that makes difficult the comparative interpretation of the data obtained (see also Ref. 1). As earlier, the greatest difference between the point values of the asymmetry and excess coefficients of V_m and u are present at low wind velocities.

Above, it has been questioned whether the indirect methods can be used in estimation of $\gamma(V_\mathrm{m})$ and $\varepsilon(V_\mathrm{m})$. The analysis of vast experimental material confirms the doubt. Temporal profiles of $\hat{\gamma}(V_\mathrm{m})$ and $\hat{\gamma}(V_\mathrm{m})_k$ at the H = 92 m (partially at H = 144 m) and T_av = 60 min are shown in Fig. 4 as the characteristic example. A good level of return signals was recorded in this range during all 12 hours of observations, and intensification of wind occurred during 2 hours after the measurement start. The value of $M(u)$ at H = 92 m first increased from 1.5 to 2.8 m/s (see Fig. 1) and then varied mainly in the range 3.5–5 m/s.

Calculations of $\hat{\gamma}(V_\mathrm{m})$ and $\hat{\gamma}(V_\mathrm{m})_k$ were carried out for the same statistical ensemble of radial components of the wind velocity vector. The Eq. (9) and the values of $M(V_\mathrm{m})$ and the parameters of uv-components obtained by the "direct" method were used in calculation of $\hat{\gamma}(V_\mathrm{m})_k$. Such a calculation of $\hat{\gamma}(V_\mathrm{m})_k$ in comparison with $\hat{\gamma}(V_\mathrm{m})$, obtained directly from the current values of $V_\mathrm{m}(i)$, also allows us to more correctly estimate the efficiency of Eq. (9). For H = 92 m, Fig. 4 shows both good coincidence of the point values of $\hat{\gamma}(V_\mathrm{m})_k$ and $\hat{\gamma}(V_\mathrm{m})$, and their unacceptably great differences (up to 1.1 in maximum). Moreover, different signs of the obtained asymmetry coefficients are often observed. It follows from the figure and the analysis of other data that the use of Eq. (9) mostly leads to increase of $\hat{\gamma}(V_\mathrm{m})_k$ relative to $\hat{\gamma}(V_\mathrm{m})$. Sometimes, the contrary situation is observed. One of such cases is shown in Fig. 4 for H = 144 m, where the last value of $\hat{\gamma}(V_\mathrm{m})_k$ is less than the corresponding value of $\hat{\gamma}(V_\mathrm{m})$ by 0.8. The value of $M(u)$ in this case was 6.3 m/s, that is quite sufficient for correctness of the expansion (5), which is the basis of the indirect methods for estimation of V_m parameters. Note that the use of Eq. (10) for calculation of $\gamma(V_\mathrm{m})$ further increases the positive difference between $\hat{\gamma}(V_\mathrm{m})_k$ and $\hat{\gamma}(V_\mathrm{m})$.

2. Analysis of estimates of the parameters of the horizontal wind velocity direction

Consider the estimate of the direction θ of the mean vector of horizontal wind velocity with the components $M(V_x)$ and $M(V_y)$, which is true for two considered methods for measurements:

$$\hat{\theta} = \arctan[\hat{M}(V_y) / \hat{M}(V_x)], \quad 0 \le \hat{\theta} < 2\pi, \qquad (12)$$

where the formulas for $\hat{M}(V_x)$ and $\hat{M}(V_y)$ are given in Ref. 1. Applying the linearization method, taking into account the relations between V_x, V_y, and uv-components of the vector V_h,[1] we obtain a simple relation for standard errors in measurement of θ and the mean of v-component:

$$\sigma(\hat{\theta}) = \sigma[\hat{M}(v)] / M(u). \qquad (13)$$

If at realization of the "direct" method,[7] the sampling mean value over N current readings of $v(i)$ is used as the $\hat{M}(v)$ estimate, then Eq. (13) takes the form

$$\sigma(\hat{\theta}) = \sigma(v) / M(u)\sqrt{N}.$$

And if Eq. (3) is true, then $\sigma(\hat{\theta}) = \sigma(\varphi) / \sqrt{N}$, that corresponds to the classic linear statistics formula for the standard error of the sampling mean value. The last relation is true for the indirect method as well, if the sampling mean values are used as the estimates of the mean values of all three radial components of

$V_r(i)$, and the number of their readings is the same and equal to N.

Consider the estimates of other characteristics of the direction φ of the vector V_h. All central moments of φ should be determined relative to the mean direction $M(\varphi)$ of the "momentary" vectors $V_h(i)$, which in general case does not coincide with the direction of the mean vector $M(V_h)$, i.e., $M(\varphi) \neq \theta$. The magnitude of this shift can be estimated through taking into account the corresponding square terms in the expansion (2). In this case, instead of Eq. (1) we obtain

$$\theta' = v / M(u) - vu' / M^2(u). \qquad (14)$$

Whence it immediately follows that

$$M(\theta') = -\mathrm{cov}(u,v) / M^2(u).$$

Thus, the difference of $M(\varphi)$ of the "momentary" vectors $V_h(i)$ from θ of the mean vector $M(V_h)$ is proportional to the correlation moment between uv-components and inversely proportional to the square of the mean values of the wind velocity $M(u)$. If the state of the wind field V_h is close to isotropic, then the correlation between different components of the wind velocity is practically absent,[8] and the aforementioned difference can be ignored. According to the Volna-3 sodar data, this deviation is truly small in the majority of events, and it is significant mainly at small wind velocities near the ground surface.

When using the indirect method of measurements, the estimate of $M(\varphi)$ can be presented in the form:

$$\hat{M}(\varphi)_k = \hat{\theta}_k + \hat{M}(\theta')_k = \hat{\theta} - \hat{\mathrm{cov}}(u,v) / \hat{M}^2(u),$$

where, according to Ref. 1,

$$\hat{\mathrm{cov}}(u,v) = \sum_{r=1}^{3} u_r v_r \hat{D}(V_r);$$

u_r, v_r are the coefficients of transition from the radial components V_r of the vector V to uv-components of the vector V_h, $\hat{D}(V_r)$ are the estimates of the variance of V_r in each channel. Then the following formula holds

$$\sigma[\hat{M}(\varphi)_k] = \sqrt{D(\hat{\theta}) + D[\hat{M}(\theta')_k] + 2\mathrm{cov}[\hat{\theta}, \hat{M}(\theta')_k]},$$

where

$$\mathrm{cov}[\hat{\theta}, \hat{M}(\theta')_k] = -2M(\theta') \times$$

$$\times \mathrm{cov}[\hat{M}(u), \hat{M}(v)] / M^2(u) - \mathrm{cov}[\hat{M}(u), \hat{D}(v)] / M^3(u).$$

$$D[\hat{M}(\theta')_k] =$$

$$= \mathrm{cov}[\hat{D}(u), \hat{D}(v)] / M^4(u) + 4M^2(\theta')D[\hat{M}(u)] / M^2(u) +$$

$$+ 4M(\theta')\mathrm{cov}[\hat{D}(u), \hat{M}(v)] / M^3(u).$$

The formulas for $D[\hat{M}(u)]$ and covariances of the estimates of the respective moments of uv-components are given in Refs. 1 and 4 or immediately follow from them. It follows from these relationships that the standard error in measuring $M(\varphi)$ is mainly determined by the random error $\sigma(\hat{\theta})$.

The estimate of the standard angular deviation $\sigma(\varphi)$ of the vector V_h, corresponding to Eq. (3), i.e., the linear part of the expansion (1) for θ', has the form

$$\hat{\sigma}_l(\varphi)_k = \hat{I}_{vk} = \hat{\sigma}(v) / \hat{M}(u).$$

Thereof the formula for the value of its random error follows:

$$\sigma[\hat{\sigma}_l(\varphi)_k] = \sigma(\hat{I}_{vk}).$$

At the same time, averaging Eq. (14) and assuming that $\mathrm{cov}^2(u,v)$, $M(u)\mathrm{cov}(u,v^2)$, $\mathrm{cov}(u'^2,v^2)$ **Z** $\sigma^2(u)\sigma^2(v)$ are true, we obtain more accurate analog of Eq. (3)

$$\sigma(\varphi) = I_v\sqrt{1 + I_u^2},$$

where I_u is the magnitude of the turbulence intensity for u-component.[3–5] So, the following estimate instead of $\hat{\sigma}_l(\varphi)_k$ can be recommended

$$\hat{\sigma}(\varphi)_k = \hat{I}_{vk}\sqrt{1 + \hat{I}_{uk}^2},$$

where $\hat{I}_{uk} = \hat{\sigma}(u) / \hat{M}(u)$.[4] But the standard error of $\hat{\sigma}(\varphi)_k$ increases due to introducing an additional *a priori* uncertainty

$$\sigma[\hat{\sigma}(\varphi)_k] = [(1 + I_{uk}^2)D(\hat{I}_{vk}) +$$

$$+ I_{uk}^2 I_{vk}^2 D(\hat{I}_{uk}) / (1 + I_{uk}^2) + 2I_{uk}I_{vk}\mathrm{cov}(\hat{I}_{uk}, \hat{I}_{vk})]^{1/2},$$

where

$$\mathrm{cov}(\hat{I}_{uk}, \hat{I}_{vk}) = \frac{1}{M^2(u)}\left\{\frac{\mathrm{cov}[\hat{D}(u), \hat{D}(v)]}{4\sigma(u)\sigma(v)} + I_u I_v D[\hat{M}(u)] -\right.$$

$$\left. - \frac{I_v \mathrm{cov}[\hat{M}(u), \hat{D}(u)]}{2\sigma(u)} - \frac{I_u \mathrm{cov}[\hat{M}(u), \hat{D}(v)]}{2\sigma(v)}\right\}.$$

This formula is obtained after linearization of the estimates $\hat{I}_{uk}$, $\hat{I}_{vk}$ and performing the required averaging. The relationships for the variances[4] $D(\hat{I}_{uk})$, $D(\hat{I}_{vk})$ follow from it, because by definition

$$\mathrm{cov}(\hat{I}_{uk}, \hat{I}_{uk}) = D(\hat{I}_{uk}) \text{ and } \mathrm{cov}(\hat{I}_{vk}, \hat{I}_{vk}) = D(\hat{I}_{vk}).$$

Application of the "direct" method for measurement of φ parameters is based on the preliminary calculation of the current directions of the vectors $V_h(i)$:

$$\varphi(i) = \arctan[V_y(i) / V_x(i)], \quad 0 \le \varphi < 2\pi,$$

and further use of methods of circular statistics.[9] These methods are based on the characteristic function[9] of the random angle φ, i.e., on the sequence of trigonometric moments τ_p relative to the zero direction $\varphi \equiv 0 \pmod{2\pi}$:

$$\tau_p = M\{\exp(jp\varphi)\} = \alpha_p + j\beta_p = \rho_p \exp(j\mu_p),$$
$$p = \pm 1, \pm 2, \ldots,$$

where

$$\alpha_p = \alpha_p(0) = M\{\cos p\varphi\},\ \beta_p = \beta_p(0) = M\{\sin p\varphi\}$$

are the cosine and sine moments of the order p;

$$\rho_p = \sqrt{\alpha_p^2 + \beta_p^2},\ 0 \le \mu_p = \arg\tau_p < 2\pi$$

are the absolute value and the polar angle of the complex number τ_p. At $\tau_1 \ne 0$, the circular mean of the random angle φ is determined unambiguously:

$$\mu = \mu_1 = M_c(\varphi) = \arg\tau_1.$$

The circular variance of the directions φ is characterized by parameter $D_c(\varphi) = 1 - \rho$, where $\rho = \rho_1 = |\tau_1|$ is the mean wind velocity with the coordinates $\{\cos\varphi(i), \sin\varphi(i)\}$ and $0 \le D_c(\varphi) \le 1$. As the circular standard deviation, the following expression is used[9]

$$\sigma_c(\varphi) = \sqrt{-2\ln[1 - D_c(\varphi)]},$$

which, in general case, varies within the range $[0, \infty]$. If $\rho > 0$ and $\varphi_0 \equiv \mu \pmod{2\pi}$, then we pass to the central trigonometric moments:

$$\alpha_p(\mu) = M\{\cos p(\varphi - \mu)\},\ \beta_p(\mu) = M\{\sin p(\varphi - \mu)\}.$$

For symmetric distribution on the circle $W(\varphi)$ $\beta_2(\mu) = 0$, therefore, it is recommended[9] to use the following formula as the asymmetry coefficient

$$\gamma_{1c}(\varphi) = \beta_2(\mu) / D_c^{3/2}(\varphi).$$

Its application is proved also by the fact that (at small variations δ of the random angle φ) $W(\varphi)$ is close to distribution on the corresponding interval of the straight line. At the same time, the results of the performed modeling and experimental investigations show that in order to gain a greater agreement between circular and linear asymmetry coefficients, it is expedient to use the normalization $\sigma_c^3(\varphi)$ in $\gamma_c(\varphi)$. Further take into account that at small δ $\sigma_c^2(\varphi) \approx 2D_c(\varphi)$ (Ref. 9) and, as distinct from Ref. 9, we use not mathematical, but meteorological definition of $\varphi(i)$. As a result, we come to the following definition of the circular asymmetry coefficient: $\gamma_c(\varphi) = -\gamma_{1c}(\varphi)/2\sqrt{2}$. And we use $\varepsilon_c(\varphi) = \varepsilon_{1c}(\varphi) + 3$ as the excess coefficient, where, according to Ref. 9, but at our normalization

$$\varepsilon_{1c}(\varphi) = \{\alpha_2(\mu) - [1 - D_c(\varphi)]^4\} / 2D_c^2(\varphi).$$

The estimates of the aforementioned circular parameters φ are based on the sampling trigonometric moments relative to the given direction φ_0 (Ref. 9)

$$\hat{\alpha}_p(\varphi_0) = a_p(\varphi_0) = \frac{1}{N}\sum_{i=1}^{N}\cos p[\varphi(i) - \varphi_0],$$
$$\hat{\beta}_p(\varphi_0) = b_p(\varphi_0) = \frac{1}{N}\sum_{i=1}^{N}\sin p[\varphi(i) - \varphi_0]. \quad (15)$$

Then the estimate of the circular mean direction has the form

$$\hat{\mu} = \hat{M}_c(\varphi) = \arctan[b_1(0)/a_1(0)],\ 0 \le \hat{\mu} < 2\pi.$$

Its standard error is obtained[9] by the linearization method, i.e., actually at the use of the expansion $\hat{\mu} = \mu + \alpha[b_1(0) - \beta]/\rho^2 - \beta[a_1(0) - \alpha]/\rho^2$ (here and below, the anomalous cases $\rho = 0$, $\rho = 1$ are not considered; $\alpha = \alpha_1$, $\beta = \beta_1$; an independence of the readings $\varphi(i)$ is also assumed), and taking into account the unbiasedness of statistics (15) relative to the initial definitions $\alpha_p(\varphi_0)$, $\beta_p(\varphi_0)$

$$\sigma[\hat{M}_c(\varphi)] = \sigma[\hat{\mu}] = \sqrt{[\rho^2 - \alpha_2(\alpha^2 - \beta^2) - 2\alpha\beta\beta_2]/2N\rho^4}.$$

The circular standard deviation can be written in the form

$$\hat{\sigma}_c(\varphi) = \sqrt{-2\ln r},$$

where

$$r = \hat{\rho} = \sqrt{a_1^2(0) + b_1^2(0)}$$

is the absolute value of the mean vector with random coordinates $\{\cos\varphi(i), \sin\varphi(i)\}$. Applying the linearization method and taking into account the formulas for variances $D[a_1(0)]$, $D[b_1(0)]$ and covariances cov $[a_1(0), b_1(0)]$ (Ref. 9), we obtain

$$\sigma[\hat{\sigma}_c(\varphi)] =$$
$$= \sqrt{[\rho^2(1 - 2\rho^2) + \alpha_2(\alpha^2 - \beta^2) + 2\alpha\beta\beta_2]/2N\rho^4\sigma_c^2(\varphi)}.$$

Write the estimate of the circular asymmetry in the form

$$\hat{\gamma}_c(\varphi) = -\hat{\gamma}_{1c}(\varphi)/2\sqrt{2}.$$

Then the following formula is true:

$$\sigma[\hat{\gamma}_c(\varphi)] = \sigma[\hat{\gamma}_{1c}(\varphi)]/2\sqrt{2}.$$

Consider two variants of estimating $\hat{\gamma}_{1c}(\varphi)$. In the first we ignore fluctuations of the sampling $\hat{\mu}$ relative to the true value $\mu = M_c(\varphi)$, i.e., actually we assume that the circular mean direction is known:

$$\hat{\gamma}_{1c}(\varphi;\mu) = b_2(\mu)/(1 - r)^{3/2}.$$

If the aforementioned fluctuations $\hat{\mu}$ are not ignored, then

$$\hat{\gamma}_{1c}(\varphi;\hat{\mu}) = b_2(\hat{\mu}) / (1-r)^{3/2}.$$

The form of relationships for the standard estimates of the first and second considered estimates is the same:

$$\sigma[\hat{\gamma}_{1c}(\varphi;\cdot)] = \left\{\frac{1}{(1-\rho)^3} D[b_2(\cdot)] + \right.$$

$$\left. + \frac{9\gamma_{1c}^2(\varphi)}{4(1-\rho)^2} D[r] + \frac{3\gamma_{1c}(\varphi)}{(1-\rho)^{5/2}} \mathrm{cov}[b_2(\cdot), r]\right\}^{1/2},$$

where

$$D[r] = [\rho^2(1-2\rho^2) + \alpha_2(\alpha^2 - \beta^2) + 2\alpha\beta\beta_2] / 2N\rho^2$$

is obtained[9] using the expansion

$$r = \rho + \beta[b_1(0) - \beta] / \rho + \alpha[a_1(0) - \alpha] / \rho.$$

It follows for $\sigma[\hat{\gamma}_{1c}(\varphi;\mu)]$ from definitions of the central moments and the estimates (15) that

$$D[b_2(\mu)] = [1 - \alpha_4(\mu) - 2\beta_2^2(\mu)] / 2N.$$

The following relation is also true

$$\mathrm{cov}[b_2(\mu), r] = \cos 2\mu\, \mathrm{cov}[b_2(0), r] -$$

$$-\sin 2\mu\, \mathrm{cov}[a_2(0), r].$$

Then, using the aforementioned expansion for r, we obtain

$$\mathrm{cov}[b_2(0), r] = (\alpha\beta_3 + 2\alpha\beta - \alpha_3\beta - 2\beta_2\rho^2) / 2N\rho,$$

$$\mathrm{cov}[a_2(0), r] = (\alpha^2 - \beta^2 + \alpha\alpha_3 + \beta\beta_3 - 2\alpha_2\rho^2) / 2N\rho.$$

The formula for $\sigma[\hat{\gamma}_{1c}(\varphi;\hat{\mu})]$ is added by the terms attributed to uncertainty of the position of the selected circular direction $\hat{\mu}$. Thus,

$$D[b_2(\hat{\mu})] = D[b_2(\mu)] + 4\alpha_2(\mu)\{\alpha_2(\mu)\sigma^2[\hat{\mu}] +$$

$$+\sin 2\mu\, \mathrm{cov}[a_2(0), \hat{\mu}] - \cos 2\mu\, \mathrm{cov}[b_2(0), \hat{\mu}]\}$$

and

$$\mathrm{cov}[b_2(\hat{\mu}), r] = \mathrm{cov}[b_2(\mu), r] - 2\alpha_2(\mu)\mathrm{cov}[\hat{\mu}, r],$$

where, using the expansion for $\hat{\mu}$ and r, we obtain the relationships for covariance terms

$$\mathrm{cov}[a_2(0), \hat{\mu}] = (\alpha\beta_3 - 2\alpha\beta - \alpha_3\beta) / 2N\rho^2,$$

$$\mathrm{cov}[b_2(0), \hat{\mu}] = (\alpha^2 - \beta^2 - \alpha\alpha_3 - \beta\beta_3) / 2N\rho^2,$$

$$\mathrm{cov}[\hat{\mu}, r] = [\beta_2(\alpha^2 - \beta^2) - 2\alpha\alpha_2\beta] / 2N\rho^3.$$

Analogously to the stated above, consider two variants of estimation of the circular excess coefficient:

$$\hat{\varepsilon}_{1c}(\varphi;\mu) = [a_2(\mu) - r^4] / 2(1-r)^2$$

and

$$\hat{\varepsilon}_{1c}(\varphi;\hat{\mu}) = [a_2(\hat{\mu}) - r^4] / 2(1-r)^2.$$

The structure of the formulas for the standard errors of both estimates is also the same:

$$\sigma[\hat{\varepsilon}_c(\varphi;\cdot)] = \left\{D[a_2(\cdot)] + 16[\varepsilon_{1c}(1-\rho) - \rho^3]^2 D[r] + \right.$$

$$\left. + 8[\varepsilon_{1c}(1-\rho) - \rho^3]\mathrm{cov}[a_2(\cdot), r]\right\}^{1/2} / 2(1-\rho)^2,$$

where the following terms correspond to $\sigma[\hat{\varepsilon}_c(\varphi;\mu)]$

$$\mathrm{cov}[a_2(\mu), r] = \cos 2\mu\, \mathrm{cov}[a_2(0), r] + \sin 2\mu\, \mathrm{cov}[b_2(0), r]$$

and

$$D[a_2(\mu)] = [1 + \alpha_4(\mu) - 2\alpha_2^2(\mu)] / 2N.$$

For $\sigma[\hat{\varepsilon}_c(\varphi;\hat{\mu})]$ we have:

$$\mathrm{cov}[a_2(\hat{\mu}), r] = \mathrm{cov}[a_2(\mu), r] + 2\beta_2(\mu)\mathrm{cov}[\hat{\mu}, r],$$

$$D[a_2(\hat{\mu})] = D[a_2(\mu)] + 4\beta_2(\mu)\{\beta_2(\mu)\sigma^2[\hat{\mu}] +$$

$$+\cos 2\mu\, \mathrm{cov}[a_2(0), \hat{\mu}] + \sin 2\mu\, \mathrm{cov}[b_2(0), \hat{\mu}]\}.$$

Note that the circular parameters of the direction of horizontal wind velocity can be determined based on the sampling trigonometric moments of the angles θ' [Eq. (2)]. The calculated values of the parameters and their standard errors coincide with those obtained through the use of the statistics φ. It is only necessary to take into account that $M_c(\theta')$ characterizes the deviation of $M_c(\varphi)$ from the direction θ of the mean vector $M(\mathrm{V_h})$. For further comparison, also consider the approach implying transfer of the initial angular distribution $W(\theta')$ from the circle to the interval of the straight line $-\pi \le \theta' \le \pi$. Then, using the linear statistics methods (see, for example, Ref. 7) we obtain new estimates for the considered angular parameters: $\hat{M}_{cl}(\theta')$, $\hat{\sigma}_{cl}(\varphi)$, $\hat{\gamma}_{cl}(\varphi)$, $\hat{\varepsilon}_{cl}(\varphi)$.

Experimental results

Figures 5–8 are constructed for the same time, place of measurements, and T_{av} as Figs. 1–3. It follows from Fig. 5 that the measurements of the mean direction of wind velocity by indirect ($\hat{M}(\varphi)_k$) and "direct" ($\hat{M}_c(\varphi)$) methods practically coincide. The sharp variations of $M(\varphi)$ at small heights can be explained by inhomogeneities of the underlying surface in the region of the experiments. Since wind velocities at these heights were insignificant, determination of their directions was accompanied by the enhanced confidence intervals. The greatest deviations of $M(\varphi)$ from θ of the mean vector of horizontal wind velocity was also observed, reaching $-7.5°$ in maximum.

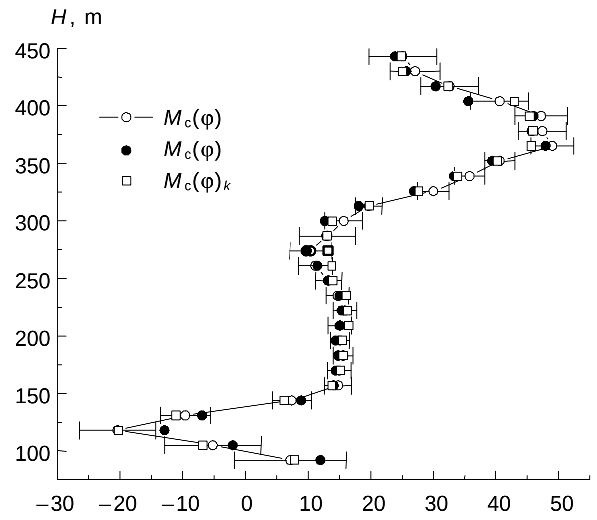

Fig. 5. Mean directions of the horizontal wind velocity, degrees.

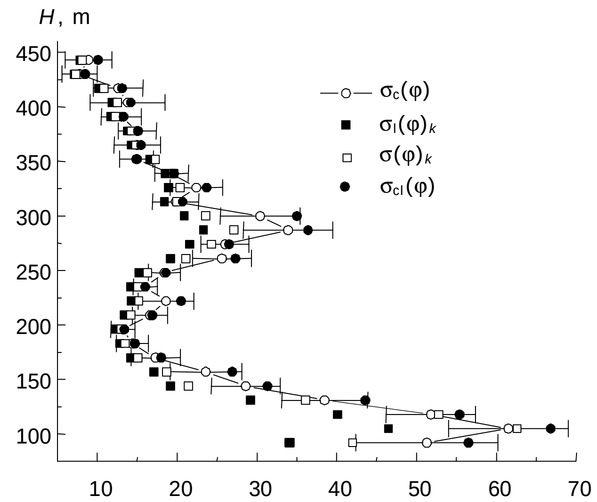

Fig. 6. Standard angular deviations of the horizontal wind velocity, degrees.

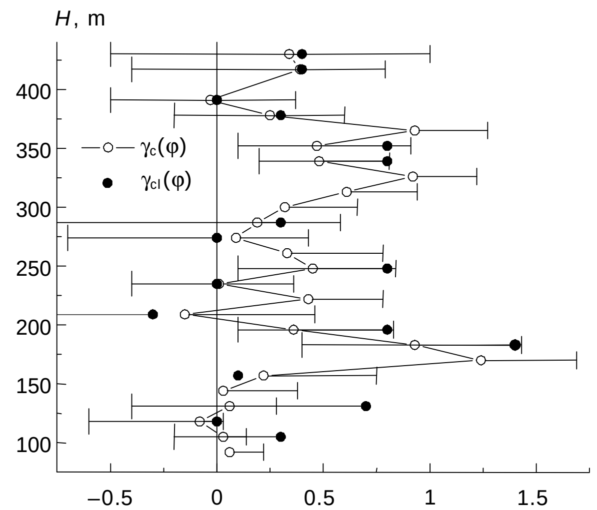

Fig. 7. Angular coefficient of asymmetry.

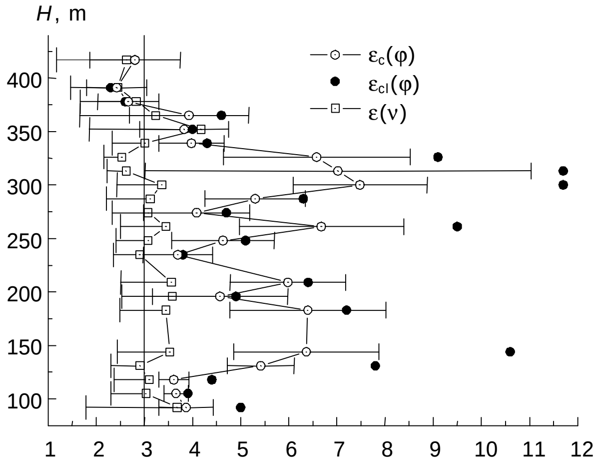

Fig. 8. Angular coefficient of excess.

The measurements of $\sigma(\varphi)$ of the vector V_h by two indirect methods ($\hat\sigma_l(\varphi)_k$, $\hat\sigma(\varphi)_k$), "direct" $\hat\sigma_c(\varphi)_k$, and also $\hat\sigma_{cl}(\varphi)_k$ are shown in Fig. 6. Note a high correlation of the obtained data, as well as the fact that $\hat\sigma_l(\varphi)_k$ always gives minimal $\sigma(\varphi)$ at all heights, and $\hat\sigma_{cl}(\varphi)$ gives maximal values. As expected, quite high $\sigma(\varphi)$ are observed at small V_m, especially near the underlying surface. Maximal scatter of the $\sigma(\varphi)$ estimates is also observed. Especially, the $\hat\sigma_l(\varphi)_k$ values are underestimated relative to three other estimates. At the same time, the $\hat\sigma(\varphi)_k$ values much better correspond to the circular standard deviations of $\hat\sigma_c(\varphi)$. Hence, at small wind velocities, approximately, up to 3–4 m/s, it is expedient to use $\hat\sigma(\varphi)_k$ based on accounting for the square terms of the expansion θ' [Eq. (14)] instead of $\hat\sigma_l(\varphi)_k$. A good correspondence between $\hat\sigma_c(\varphi)$ and $\hat\sigma_{cl}(\varphi)$ turned out to be unexpected. The confidence intervals $I_{0.9}$ for $\hat\sigma_c(\varphi)$ at all heights cover the point $\hat\sigma_{cl}(\varphi)$ values. The noted correspondence is confirmed by the conducted investigations with a great amount of experimental data at different T_{av}. Therefore, in principle, the standard (not circular) sampling deviation of the initial angles $\theta'(i)$ [Eq. (2)] can be used for determination of $\sigma(\varphi)$. Although, the values of $\hat\sigma_{cl}(\varphi)$ and their $I_{0.9}$ always will be somewhat overestimated relative to $\hat\sigma_c(\varphi)$ values. The differences between all noted estimates of $\sigma(\varphi)$ decrease as V_m increases.

The results of measuring the angular asymmetry coefficient are shown in Fig. 7

$$\hat\gamma_c(\varphi) = -\hat\gamma_{1c}(\varphi;\hat\mu)/2\sqrt{2} \text{ and } \hat\gamma_{cl}(\varphi).$$

The majority of these point values, as $\hat\gamma(V_m)$ in Fig. 3, are positive. But, as opposite to measurements of $\sigma(\varphi)$, no agreement between them is observed, and

$I_{0.9}[\hat{\gamma}_{cl}(\varphi)]$ values essentially exceed $I_{0.9}[\hat{\gamma}_c(\varphi)]$. However, these measurements presented in the interval form, in principle, do not contradict to each other. Thus, all $I_{0.9}[\hat{\gamma}_c(\varphi)]$ cover the corresponding $\hat{\gamma}_c(\varphi)$ values. This is also true for evidently doubtful point values of $\hat{\gamma}_{cl}(\varphi)$ with $I_{0.9} \geq 2.5$, which are not shown in Fig. 7. On the whole, we can state that the use of $\hat{\gamma}_{cl}(\varphi)$ is inexpedient in practice. Analogous conclusion for $\hat{\varepsilon}_{cl}(\varphi)$ follows from Fig. 8, because the differences between $\hat{\varepsilon}_{cl}(\varphi)$ and $\hat{\varepsilon}_c(\varphi) = \hat{\varepsilon}_{1c}(\varphi;\hat{\mu}) + 3$ can be quite great. This is also true for the indirect estimate $\hat{\varepsilon}(\varphi)_k = \hat{\varepsilon}(v)$, which follows from Eq. (4), i.e., from the linear part of the expansion θ' [Eq. (2)]. Taking into account the square terms as in calculation of $\hat{\sigma}(\varphi)_k$ turned to be inefficient. Note great values of the circular excesses in the middle part of the height range, that is evidence of sharp peaks in the corresponding angular distribution $W(\varphi)$ and their significant differences from the wrapped normal distribution and the Mises distribution, which play a central role in circular statistics. The data obtained with $\hat{\varepsilon}(v)$, which, in the strong meaning, corresponds to $W(v)$ on the straight line, contradict to this conclusion. Similarly to the distribution on the straight line, the estimation of large circular $\varepsilon(\varphi)$ is accompanied by breaks of their profiles and high values of $I_{0.9}$ (Ref. 1).

Summarizing all stated above, we can ascertain that the use of indirect methods for estimating $\gamma(V_m)$, $\varepsilon(V_m)$, $\gamma(\varphi)$, $\varepsilon(\varphi)$ in acoustic sensing of the atmosphere at the characteristic spatial-temporal scales of averaging and selection of data, can lead to quite essential uncontrolled errors. Therefore, only "direct" method is applied in the Volna-3 sodar for determination of the asymmetry and excess coefficients of the horizontal wind velocity and direction. Using the formulas presented in this paper, one can estimate the degree of uncertainty of the parameters V_m and φ measured by the sodar, that allows us to interpret more correctly the results of acoustic sensing of the atmosphere.

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