Orientation of particles in *Ci* crystal clouds. Part 2. Azimuth orientation

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Received September 27, 2005

We discuss the possibility that particles of a crystal cloud can take a preferred orientation with respect to wind direction under the action of aerodynamic forces in the atmosphere. It is shown theoretically that the preferred orientation can be caused by wind speed pulsations and by the forces applied to a particle falling due to gravity in the presence of wind speed gradient. The theoretical estimate of the parameter of the function of particle orientation distribution over azimuth angles well agrees with its experimental estimate from the backscattering phase matrices of cirrus clouds obtained from data of polarization lidar measurements.

Introduction

The effect of air turbulence on angular distribution of orientation of the maximum diameter of particles about the horizontal direction was considered in the first part of the paper.[1] Such an orientation appears under the effect of aerodynamic forces on large non-spherical particles falling down due to gravity. In this paper we discuss the possibility that particles of a crystal cloud can take orientation along a preferred azimuth direction due to aerodynamic forces. This means that particles can take the position when their characteristic sizes (for certainty the maximum diameters) are grouped about some azimuth direction in the horizontal plane.

Probably, the first mention of the possibility of such an orientation came from the observations of inclined sun pillars, quite a rare phenomenon.[2] Nevertheless, the deviation of the sun pillar from the vertical direction up to 10° was observed in some cases. The author of Ref. 2 supposed that the inclination of the pillars is caused by the wind displacement of particles.

We have obtained direct indications of the presence of azimuth orientation of particles in backscattering phase matrices obtained from polarization lidar measurements.[3] Manifestations of the azimuth orientation were observed quite often in our measurements. However, in the majority of cases the orientation was only poorly pronounced. We supposed that the pulsations of wind velocity are responsible for the azimuth orientation. As the moments orienting a particle are proportional to the square of the difference between the velocities of the particle and the airflow, one can assume that the pulsed motions of particles can cause their orientation. It is *a priori* clear that the particles should not be completely entrained by the pulsed motions of air, and these motions should have different characteristics in different directions. In other words, orientation of particles should be expected from the side of pulsations belonging to the range of anisotropic turbulence keeping the orientation along wind. The purpose of this paper is to justify the validity of this hypothesis by means of quantitative estimates presented below.

Equations of motion

The force proportional to the difference between the velocities of the particle and the airflow affects the particle moving together with the airflow. The equation of motion has the following form:

$$m\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \kappa D\eta(\mathbf{v} - \mathbf{u}) - mg, \qquad (1)$$

where m is the particle mass, D is the particle diameter, η is the viscosity of air; $\mathbf{v}$ and $\mathbf{u}$ are the velocities of the flow and the particle, respectively, g is the acceleration due to gravity. The factor κ depends on the particle shape. For a sphere $\kappa = 3\pi$. For oblate and elongated ellipsoids with the semi-axis ratios 1/2.5 $\kappa \cong 2.5\pi$ and 2π, respectively. Here D means the length of the maximum diameter.

Let us consider the equation describing the behavior of the x-component assuming the x-axis to be directed horizontally along the wind direction, and z-axis along vertical direction

$$\frac{\mathrm{d}u_x}{\mathrm{d}t} + \omega_0 u_x = \omega_0\left(\langle v_{0,x}\rangle + \frac{\partial\langle v_x\rangle}{\partial z}u_z t + v'_x\right). \qquad (2)$$

The first term in parentheses of the right-hand part of this equation denotes the mean wind speed at a height z_0, the second term describes the increase of the mean wind speed in the presence of vertical gradient of the wind velocity, and the third term is its pulsation component. Let us then assume that

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$\partial v_x/\partial z$ = const. The value u_z means the velocity of the particle falling down, which is also assumed constant. The factor $\omega_0 = \kappa D\eta/m$ for oblate and elongated ellipsoids is equal, respectively, to

$$\omega_0 = 15\eta/\rho_p D^2\sqrt{1-e^2}, \quad \omega_0 = 12\eta/\rho_p D^2(1-e^2), \quad (3)$$

where ρ_p is the density of the particle substance, and e is the ellipsoid eccentricity.

Since the differential equation (2) is linear, one can present its solution as a sum of solutions of the equations

$$\frac{du_x}{dt} + \omega_0 u_x = \omega_0(\langle v_{0,x}\rangle + \frac{\partial\langle v_x\rangle}{\partial z}u_z t), \quad (4)$$

$$\frac{du_x}{dt} + \omega_0 u_x = \omega_0 v'_x. \quad (5)$$

Solution of Eq. (4) has the following form:

$$u(t) = \langle v_0\rangle[1-\exp(-t/\tau_0)] + Gt - G\tau_0[1-\exp(-t/\tau_0)], \quad (6)$$

with the initial conditions

$$u(t=0) = 0, \quad \langle v(z_0, t=0)\rangle = \langle v_0\rangle,$$

where $G = u_z(\partial\langle v_x\rangle/\partial z)$, $\tau_0 = 1/\omega_0$.

Solution of Eq. (6) has simple physical meaning. At the time $t \gg \tau_0$ the speed of particle, depending on the sign of $\partial\langle v_x\rangle/\partial z$, is below or above the value of the mean wind speed $\langle v_0 + Gt\rangle$, by the constant value $G\tau_0$ and is permanently affected by the airflow. Later on, we shall consider its effect on the particle orientation.

Solution of the equation (5) for sinusoidal pulsations $v'(t) = A\sin\omega t$ has the form[14]

$$u(t) = \frac{A\omega\tau_0\exp(-t/\tau_0)}{1+\omega^2\tau_0^2} + \frac{A\sin(\omega t-\varphi)}{\sqrt{1+\omega^2\tau_0^2}}, \quad (7)$$

where $\tau_0 = 1/\omega_0$, $\varphi = \arctan\omega\tau_0$.

The first term characterizes the transition process, while the second one the steady harmonic motion. The amplitude of this motion is less than the amplitude of pulsations of air and is behind them in phase. Further, the mean square of the difference of the velocities of pulsations of the air and a particle at a steady motion will be of interest

$$\langle(\Delta u)^2\rangle = \langle(v'-u)^2\rangle = \langle A^2[\sin\omega t - \sin(\omega t-\varphi)/\sqrt{1+\omega^2\tau_0^2}]^2\rangle. \quad (8)$$

Table 1. Calculated ratio $\langle\Delta u^2\rangle/A^2$, as a function of $\omega\tau_0$

$\omega\tau_0$	0.1	0.2	0.4	0.5	0.7	1.0	5.0	10	100
$<\Delta u^2>/A^2$	0.01	0.02	0.07	0.105	0.166	0.25	0.481	0.495	0.499

As is seen in the first column of the Table 1, at $\omega \le 0.1/\tau_0$ the square of the difference of the amplitudes of pulsations is only one percent or less of the square of the amplitudes of pulsations of the air. This reflects the fact that in the case of low-frequency pulsations the aerosol is completely entrained admixture. The values of the characteristic time for the particles of different size are shown in Table 2.

Table 2. The characteristic time $\tau_0 = 1/\omega_0$ (s), determined by Eq. (3), at different maximum diameters of particles D

Particle shape	D, μm							
	10	50	100	200	400	600	800	1000
plates	$4\cdot10^{-4}$	$4\cdot10^{-3}$	0.02	0.15	0.28	0.64	1.14	1.78
columns	$3\cdot10^{-4}$	$2\cdot10^{-3}$	0.01	0.07	0.14	0.32	0.57	0.89

Orientation by wind pulsations

Let us determine the lower boundary of the effective frequencies of pulsations using the data presented. Taking $\tau_0 = 2$ s, from the condition $\omega\tau_0 \ge 0.1$ we obtain that $\omega_l \ge 0.05$ rad/s. At lower frequencies, even the largest particles are entrained by the airflow. Assuming that isotropic turbulence can not orient particles, let us determine the upper boundary of the effective frequency of the Taylor microscale[5,6]

$$\omega_T = \sqrt{\varepsilon/15\nu}, \quad (9)$$

where ε is the rate of dissipation of energy, $m^2\cdot s^{-3}$; ν is the kinematic viscosity of air, which we assume to be equal to $3\cdot10^{-5}$ m^2/s for the temperatures at the heights of 10 km.

The rate of dissipation varies within wide limits from $5\cdot10^{-4}$ in the calm atmosphere to $5\cdot10^{-1}$ $m^2\cdot s^{-3}$ in cumulus clouds.[6,7] Then the Taylor frequency ω_T varies within the range from 2 to 15 rad/s.

For the elementary volume of air moving together with the airflow, the variance of the velocity is equal to (Ref. 8)

$$\langle v'^2\rangle = \varepsilon/\omega. \quad (10)$$

Assuming that in Eq. (8) $\langle A^2\rangle = \varepsilon/2\omega$, at a preset τ_0, i.e., the particle size is also preset, let us write the formula for the square of the difference of the pulsation velocities $\langle(\Delta u)^2\rangle$, averaged over a significant frequency range $[\omega_l, \omega_T]$, with the weighting function $1/\omega$ and over the time interval $T \gg \tau_0$

$$\langle(\Delta u)^2\rangle = \frac{1}{2T}\int_0^T dt\int_{\omega_l}^{\omega_T}\varepsilon\omega^{-2}[\sin\omega t - \sin(\omega t-\varphi)/\sqrt{1+\omega^2\tau_0^2}]^2 d\omega. \quad (11)$$

Let us substitute the obtained value into the formula for the moment of force, which makes the particle to turn maximum diameter across to the direction of motion[9,10]

$$M(\varphi) = \lambda\langle(\Delta u)^2\rangle\rho V\sin 2\varphi/2, \quad (12)$$

where φ is the angle between the direction of the velocity of airflow and the minimum axis of the ellipsoid of revolution, ρ is the air density, V is the ellipsoid volume, λ is the shape factor depending on the ratio between the maximum and minimum semi-axes of the ellipsoid[10]

$$\lambda = \{e^3[e - \sqrt{1-e^2}(\frac{\pi}{2} - \arctan\frac{\sqrt{1-e^2}}{e})]^{-1} - 1\}^{-1}. \quad (13)$$

The volumes of the spheroid and elongated ellipsoid of revolution can be represented by the length of the maximum diameters D and eccentricity according to the following formulas:

$$V_s = \frac{1}{6}\pi D^3\sqrt{1-e^2},\ \ V_e = \frac{1}{6}\pi D^3(1-e^2). \quad (14)$$

Then, for determining the distribution function over orientation angle φ, let us use the same approach as that in Ref. 1. The main principles accepted there lie in the fact that the turbulence cells of the inner scale (dissipation interval) are responsible for destruction of orientation, and the efficiency of interactions of particles with these cells is proportional to the ratio of their volumes.

The scales of the length and the velocity in the dissipation interval have the following values[5,8]:

$$l_0 = (\nu^3/\varepsilon)^{\frac{1}{4}};\ \ u_0 = (\varepsilon\nu)^{\frac{1}{4}}. \quad (15)$$

The cell of the diameter l_0 has, on average, the energy

$$w = \frac{1}{6}\pi l_0^3\rho\langle u_0^2\rangle. \quad (16)$$

The energy of the cell is determined from Eqs. (15) and (16) as a function of the mean rate of dissipation

$$w(\varepsilon) = \frac{1}{6}\pi\rho\nu^{\frac{11}{4}}\varepsilon^{-\frac{1}{4}}. \quad (17)$$

Efficiency p of the energy transfer from a turbulence cell to a particle is assumed equal to the ratio of their volumes. For the elongated ellipsoids of revolution

$$p(D,\varepsilon) = D^3(1-e^2)(\nu^3/\varepsilon)^{-\frac{3}{4}}. \quad (18)$$

Kinetic energy obtained by a particle in collision is equal to

$$W(D,\varepsilon) = p(D,\varepsilon)w(\varepsilon). \quad (19)$$

The distribution over the orientation angles is determined from differential equation describing the balance of the potential energy of particles and the kinetic energy of turbulent pulsations

$$nM(\varphi,D)\mathrm{d}\varphi = -W(D,\varepsilon)\mathrm{d}n. \quad (20)$$

Integration of this equation under the normalization conditions[1] gives the following distribution over the angles of azimuth orientation about the mode φ_0 of the distribution:

$$n_D(\varphi-\varphi_0) = N_D\exp[k_D\cos 2(\varphi-\varphi_0)]/\pi I_0(k_D), \quad (21)$$

where N_D is the total number density of particles of the size D; $I_0(k)$ is the modified Bessel function of the first kind and zeroth order. The indices D emphasize that the distribution refers to particles of a certain size. The angle φ is counted from the wind direction to the direction of the minimum axis of the particle. If one considers maximum diameters of particles, the distribution (21) characterizes their grouping about the direction $\varphi_0 \pm \pi/2$.

Taking into account formulas (12) to (19), the parameter k of the distribution is determined as follows:

$$k_D(D,\varepsilon) = \lambda\langle(\Delta u_D)^2\rangle\xi/\sqrt{\nu\varepsilon}. \quad (22)$$

The additional parameter ξ is introduced in Eq. (22) as a characteristic of the anisotropy of turbulence. It depends on the ratio of the time scales in Lagrange integral for longitudinal and transverse pulsations and is introduced for taking into account the destructive action of pulsations along y-axis on the orientation of the maximum diameters across the x-axis. Pulsations along z-axis do not prevent this orientation. It is taken in our estimates that $\xi = 0.5$. The obtained estimates of the parameter k for particles of different size at three values of the energy dissipation rate are shown in Table 3. The rounded to integer values of angles the rms deviations of the particle axes directions from the mode of the distribution are also given here.

It gives some idea of the degree of orientation. Up to 60 or 70% of particles fall to the range $\pm\sigma$. The value $\sigma = 52°$ corresponds to the uniform azimuth distribution of the particle axes.

Table 3. The values of the parameter *k* of the particle orientation distribution (21) over the azimuth angle and rms deviation from the distribution mode σ for elongated ellipsoids of revolution of the length *D* and the axes ratio 1/2.5 for different energy dissipation rates ε

ε, m^2/s^3		D, μm						
		400	500	600	700	800	900	1000
10^{-3}	k	0.015	0.036	0.10	0.11	0.17	0.24	0.36
	σ°	52	51	50	50	49	48	46
10^{-2}	K	0.15	0.23	0.58	0.93	0.97	1.22	1.53
	σ°	50	48	43	38	37	33	30
10^{-1}	k	0.78	1.25	1.78	2.53	3.29	4.02	4.89
	σ°	40	33	27	21	18	16	14

It is worth noting that the obtained estimates are comparable with the experimental results presented in Ref. 3. It is shown[11,12] that the parameter determined from the experiment

$$\chi = (m_{22} + m_{33})/(m_{11} + m_{44}),$$

where m_{ii} are the elements of the normalized backscattering phase matrix, is related to the parameter k of the distribution (21) by the relationship

$$\chi(k) = I_2(k)/I_0(k),$$

where I_0 and I_2 are the modified Bessel function of the first kind and of the zeroth and second order, respectively. This dependence is shown in Fig. 1.

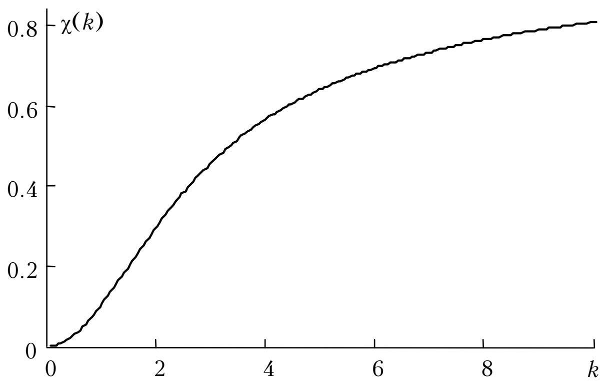

Fig. 1. Theoretical dependence of the parameter χ represented by the elements of the normalized reduced backscattering phase matrix[12] as $\chi(k) = (m_{22} + m_{33})/(1 + m_{44})$ on the parameter k of the distribution (21).

The distribution of the parameter χ obtained from a big sample of the backscattering phase matrices in presented in Ref. 3. The mode of the distribution is at the value of 0.1. It corresponds to the value $k \cong 1$. It is seen in Table 3 that for quite large particles at $\varepsilon = 10^{-2}$ the values k are of the same order of magnitude.

The maximum value χ obtained in the experiments was equal to 0.62 that corresponds to $k = 4.65$. The value in the third row of Table 3 is not very far from this value. However the presented extreme value of χ is related to a specific atmospheric situation when the directions of the azimuth orientation of particles in two cloud layers at the heights of 6.0 and 6.7 km differed by almost 90°. Such an unusual behavior caused the supposition that the wind shift occurred at these heights, which could be accompanied by strongly developed turbulence. It is clear that, in addition to the horizontal component, the vertical component of the gradient of wind velocity could also be present. One can estimate the effect of this factor on the azimuth orientation based on Eq. (6).

Orientation in the presence of the vertical gradient of the wind velocity

According to Eq. (6), in the case of a steady-state motion the difference between the mean velocities of wind and particle is

$$\delta u = \frac{\partial v_x}{\partial z} u_z \tau_0,$$

where $\tau_0 = 1/\omega_0$ is defined by Eq. (3), u_z is the velocity of the particle fall due to gravity. As in Ref. 1, it can be presented by the particle size using the empirical relationship

$$u_z = 10^{3\beta - 2} A D^{\beta},$$

where A and β are the empirical constants introduced in Ref. 13. One should substitute the value $(\delta u)^2$ into Eq. (12) instead of $(\Delta u)^2$.

The calculated results on the parameter $k(D)$ of the distribution of the form (21) are shown in Fig. 2 as functions of the particle size for three pairs of values of the vertical gradient of the wind velocity and the energy dissipation rate. It was assumed that the dissipation rate is greater at high values of the gradient. Calculations were performed for the columns with the aspect ratio of 1:2.5. The constants A and β were equal to 70 and 0.92, respectively.

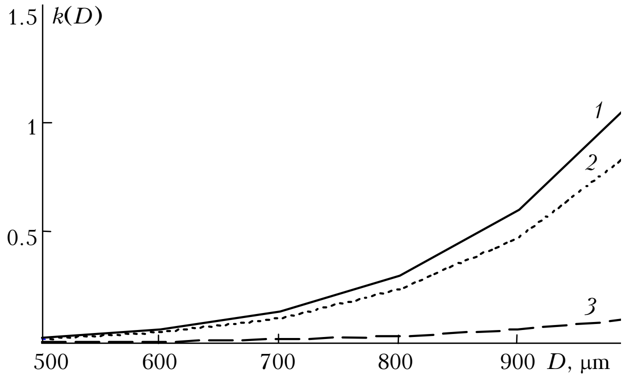

Fig. 2. Dependence of the parameter k of distribution (21) on the length of the column crystals at different values of the vertical gradient of wind velocity $\partial v_x/\partial z$ and the energy dissipation rate ε: (*1*) $\partial v_x/\partial z = 0.1\ s^{-1}$, $\varepsilon = 0.1\ m^2 \cdot s^{-3}$; (*2*) $\partial v_x/\partial z = 0.05\ s^{-1}$, $\varepsilon = 0.01\ m^2 \cdot s^{-3}$; (*3*) $\partial v_x/\partial z = 0.01\ s^{-1}$, $\varepsilon = 0.001\ m^2 \cdot s^{-3}$.

The strong dependence on the particle size ($\sim 10^6$) is seen in Fig. 2. It is clear. The parcel sedimentation velocity and the squared characteristic time approximately linearly depend on the particle size. Then the squares are taken of both parameters in order to obtain $(\delta u)^2$. Essential orientation is observed for the columns of the length greater than 800 μm and only at quite great values of the vertical gradient of the wind velocity. Fulfillment of the contradictory conditions is necessary for reaching strong orientation, namely, the presence of great gradient of the wind velocity and poorly developed turbulence (small values of the energy dissipation rate). Comparison with the results presented in Table 3 enable us to conclude that this mechanism of orientation has less efficiency than that due to pulsations of the wind velocity.

Conclusion

The aforementioned estimates confirm the possibility that particles of crystal clouds can take a preferred azimuth orientation due to pulsations of wind velocity under conditions of a well-developed air turbulence. The following assumptions were used in making the estimates. Pulsations in the frequency range, the lower boundary of which is determined by the characteristic time of the largest particles, while the upper one being determined as the Taylor microscale frequency, are responsible for the orientation. The Taylor microscale frequency depends on the energy dissipation rate and so it is variable. Turbulence in this frequency range is assumed anisotropic so that the time scales in the Lagrange integral of the longitudinal and transverse pulsations differ by approximately one order of magnitude. Turbulence is assumed isotropic for the frequencies above the boundary determined by the Taylor microscale. The cells with the size of the inner turbulence scale are assumed destructing the orientation process. The efficiency of interaction of particles with these cells is proportional to the ratio of their volumes.

The estimates obtained under the aforementioned assumptions agree with estimates of the parameter of the function of particle orientation distribution over azimuth angles obtained from lidar data on the backscattering phase matrices of crystal clouds.

Acknowledgments

The work was supported in part by the Russian Foundation for Basic Research (grant No. 04-05-64495) and Federal Agency for Science and Innovations of the Ministry of Education and Science of the Russian Federation (State contract No. RI 26/071).

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