Thermal blooming of laser beams along atmospheric paths and diagnostics of their parameters

V.E. Zuev, V.P. Aksenov, and V.V. Kolosov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk

The paper describes results of theoretical investigation of high-power laser beam propagation along the ground and high-altitude atmospheric paths as well as the potentialities of remote measurement of beam characteristics. The paper presents an overview of the algorithms for solving the problems of laser radiation blooming based on the method of statistical tests, the equation for statistical moments of a field complex amplitude and the radiation transfer equation. The parameters of partially coherent radiation along characteristic atmospheric paths are analyzed. Recommendations are given on how to optimize the radiation focusing and power. The theoretical grounds of new methods for diagnostics of spatial–power and phase characteristics of laser beams are presented. We also present some results of numerical and laboratory experiments on reconstruction of the intensity and phase distributions.

Introduction

The increasing number of the optoelectronic instruments' applications to sounding, optical detection and ranging, transportation of optical power through the atmosphere or focusing it on a high target have arisen a considerable interest of researchers in the study of high-power laser radiation propagation through the real atmosphere. In this case it is necessary to take into account the aerosol and molecular absorption, the radiation scattering by aerosols, regular variation of atmospheric parameters along the propagation path, the effect of turbulence, the geometry of the problem, and a series of other factors. Propagation of high power laser radiation can be accompanied by nonlinear effects. In such a formulation the problem of propagation of limited wave beams can be solved only by numerical methods.

On the other hand, the creation of systems for adaptive control and optimization of beam parameters calls for a search and development of new remote techniques to measure spatial, power, and phase characteristics of optical radiation.

New measurement techniques were developed when improving and, in some cases, devising novel approaches to solve ill-posed inverse problems occurring in one or the other subject areas. Because the solution of inverse problems is often based on the solution of the direct ones, the common solution of these problems is required.

1. Numerical methods of the investigation of laser radiation blooming

Numerical methods for solving the problems in studies of high-power laser beam propagation through the atmosphere have been being developed intensively since early the 70's. Those were based on the solution of the parabolic equation. First, the algorithms for axisymmetric cases were developed.[1,2] The possibility of constructing effective algorithms for solving the problems in the absence of axial symmetry relies on the use of the splitting method.[3,4]

The five-point patterns and schemes of higher orders based on the method of finite elements formed the foundation for the difference schemes.[5-8] The development of the fast Fourier transform algorithm[9,10] enabled one to construct numerical algorithms using an approach in which the linear diffraction equation is solved in the spectral space, and nonlinear refraction is taken into account in the approximation of a phase screen at each step of the propagation.[11-14]

In investigating the laser radiation propagation, it is necessary to take into account that radiation can inherently be partially coherent. It is important that even in the simplest case of radiation diffraction in vacuum the parabolic equation becomes inapplicable because the partial coherence can result not only in the increase of the beam divergence but essentially modifies intensity distribution in a focal plane. Therefore, in early the 80's the problem of the development of new theoretical methods was stated because, on the one hand, the urgency of the problem on investigating propagation of high-power laser radiation along atmospheric paths, and, on the other hand, the gap then existed between the practical needs and the development of the theory for this problem. The radiation used in laser systems was essentially incoherent (the beam divergence exceeded the diffraction-limited divergence by an order of magnitude), while the calculations were based on the solution of the parabolic equation that describes the propagation of coherent radiation. There are two classical approaches

to investigation of propagation of random (partially coherent) fields:

1) the stochastic approach based on the Monte Carlo method;

2) the method of moments.

High level of the development of numerical algorithms of the solution of parabolic equation predetermined its wide use in the investigation of dynamics of light field statistics based on the method of statistical tests.[11,15–18] At the point of entry to the medium a set of pseudorandom field realizations was simulated satisfying the statistical characteristics of a source of incoherent radiation. Then we averaged the results of multiple solution of the parabolic equation for a complex field amplitude and determined a set of power and statistical characteristics of a partially coherent beam. It should be noted that the method of statistical tests does not require supplementary limitations, typical for the study by analytical methods of randomly modulated wave propagation. But its practical application is limited by the condition that the time of radiation coherence essentially exceeds other characteristic dynamic times of the problem, in particular, the time of radiation coherence must exceed the time of nonlinear response of the medium.

That means that in the framework of this approach it is possible to consider only the so-called, narrow-band partially coherent radiation.

Real situation for high-power laser beams corresponded to the inverse condition since the time of laser beam radiation coherence did not exceed 10^{-6} s, and the time of nonlinear response is about 10^{-1} s. Thus, the only possible approach to solving this problem was the method based on solution of equations for moments of the complex field amplitude.[15,18–21]

1.1. Method of moments

The correlation theory of random waves is restricted to consideration of only the first and second moments. This enables one to study the dynamics of such quantities as the coherence time, the coherence length (radius), the degree of coherence, the distribution of mean intensity, and the Poynting vector. If it is necessary to study the dynamics of the radiation intensity fluctuations, we can use the equations for moments of higher orders.

The passage from parabolic equation for a complex field amplitude

$$2ik \frac{\partial E}{\partial z} + \nabla_R^2 E +$$

$$+ k^2 \left[\varepsilon_r (z, \mathbf{R}) + i\, \varepsilon_i (z, \mathbf{R}) \right] E(z, \mathbf{R}) = 0 \qquad (1)$$

(k is the wave number; ε_r, ε_i are the real and imaginary parts of the relative disturbance, $\Delta\varepsilon$, of the dielectric constant of a medium) to a closed equation for the coherence function (the second-order moment)

$$2ik \frac{\partial c_2}{\partial z} + 2\nabla_{\mathbf{R}}\nabla_{\rho}\Gamma_2 + k^2 \left[\Delta\varepsilon(z, \mathbf{R} + \rho/2) - \right.$$

$$\left. - \Delta\varepsilon^*(z, \mathbf{R} - \rho/2) \right] \Gamma_2 (z, \mathbf{R}, \rho) = 0 \qquad (2)$$

in the problems on thermal blooming assumes a possibility of splitting field correlators and dielectric constant, being a functional of the radiation intensity.

For the radiation, propagated in a medium with local inertial nonlinearity, a possibility of such a splitting was considered in the Ref. 22, where it was assumed that the splitting was possible if the effect of nonlinear disturbance of the dielectric constant due to variation of the field phase $\delta\varphi$ at the length of the longitudinal field correlation remained to be small. In the Ref. 18 the same problems were analyzed based on the concept that the medium is a linear system, whose transmission coefficient is determined by the constitutive equation for the dielectric constant. In this case it is possible to determine the dependence of the variance of dielectric constant fluctuations at the output of the system on the spectral density of intensity fluctuations at the system input and to define the conditions when the splitting of correlators is possible in the equation for $\Gamma_2(z, \mathbf{R}, \rho, t)$.

The fact, which is essential for these assessments, is that no dependence on the propagation distance occurs in the expressions obtained. From the results of the investigations carried out in the framework of the perturbation method[16,17,23–25] it follows that at thermal blooming the infinitesimal initial fluctuations of the wave phase and amplitude results in the exponential growth of fluctuations with the increasing distance. The other result of the studies in Refs. 16 and 17 is the conclusion drawn there on the applicability of the above equation for the coherence function only for weakly nonlinear media (the refraction parameter must not exceed 1). However, in all the above-mentioned papers the following condition was used

$$\tau_0 \gg \tau_a, \qquad (3)$$

where τ_0 is the correlation time for the radiation intensity fluctuations; τ_a is the characteristic time of a nonlinear response of the medium to the variation of radiation intensity that corresponds to the so-called narrow-band partially coherent radiation.

In Ref. 26 we have determined the applicability limits of the correlator splitting in the equation for the second order coherence function at thermal blooming of a wide-band radiation, i.e., under condition

$$\tau_0 < T \ll \tau_a, \qquad (4)$$

where T is the period of pulse sequence for the pulse-periodic mode; $\tau_a = a/v$, a is the beam radius, v is the wind velocity component transverse to the direction of beam propagation. At thermal blooming of optical radiation in a moving medium the constitutive equation for estimating nonlinear perturbation of the dielectric constant in the isobaric approximation has the form[16,23,26]:

$$\left(\frac{\partial}{\partial t} + v \,\frac{\partial}{\partial x} - \lambda_T \,\Delta_\perp\right) \tilde{\varepsilon}(z, \mathbf{r}, t) =$$

$$= \alpha \left|\frac{d\varepsilon}{dT}\right| / (\rho c_p) \, I(z, \mathbf{r}, t), \qquad (5)$$

where λ_T is the coefficient of heat conductivity of the medium; α, p, c_p are the volume absorption coefficient, density, and specific heat of the medium, respectively.

Thermal blooming of partially coherent radiation in the absence of wind ($v = 0$) was also considered. The mode of thermal blooming was investigated experimentally and theoretically in Ref. 23. There are some reasons for assuming that the time of coherence τ_0 of radiation exceeds all other characteristic times of the problem, i.e., the condition (3) is fulfilled. To account for the influence of coherence time on the dynamics of the field fluctuations, in Ref. 26 this assumption was not used, and, accordingly, the time derivative in Eq. (5) remained. To eliminate the influence of beam distortions, as a whole, on its fluctuation characteristics, we consider a beam with a uniform distribution of the mean intensity over the aperture, whose size satisfies the conditions

$$a \gg r_k, \quad a \gg \sqrt{2\pi z / k} \,, \qquad (6)$$

where r_k is the coherence radius. The field was represented in the form

$$E(z, \mathbf{r}, t) = E_0 \exp[\chi(z, \mathbf{r}, t) + iS_0(z, \mathbf{r}, t)] +$$

$$+ iS(z, \mathbf{r}, t), \qquad (7)$$

where $\chi(z, \mathbf{r}, t)$ is the logarithm of the relative field amplitude; $S_0(z, \mathbf{r}, t)$, $S(z, \mathbf{r}, t)$ are the regular and fluctuation components of the phase shift.

It was assumed that $\chi \ll 1$. Then for the Fourier transforms of fluctuations of the logarithm of amplitude χ and phase fluctuations S along transverse coordinates we can write the following set of equations[25,26]:

$$\frac{\partial \chi(z, \varkappa, t)}{\partial z} - \frac{\varkappa^2}{2k} S(z, \varkappa, t) = 0;$$

$$\frac{\partial S(z, \varkappa, t)}{\partial z} + \frac{\varkappa^2}{2k} \chi(z, \varkappa, t) +$$

$$+ 2kp_0 \int\limits_0^t \chi(z, \varkappa, t') \, e^{-(t-t')\lambda_T \varkappa^2} \, dt' = 0, \qquad (8)$$

where, $p_0 = \alpha E_0^2 / (2\rho c_p) \left|d\varepsilon / (dT)\right|$, $[p_0] = 1/s$, i.e., the rate of variation of $\Delta\varepsilon$, $\varkappa$ is the spatial frequency.

In addition to the set of Eqs. (8) the boundary conditions are presented as follows:

$$\chi(z = 0, \varkappa, t) = \chi_0(\varkappa, t); \quad S(z = 0, \varkappa, t) = S_0(\varkappa, t)$$

and if we assume that fluctuations of the level and phase are statistically independent and

$$< \chi_0(\varkappa, t) \, \chi_0(-\varkappa, t + \tau) > = F_{\chi_0}(\varkappa) \, e^{-p_k |\tau|},$$

$$< S_0(\varkappa, t) \, S_0(-\varkappa, t + \tau) > = F_{S_0}(\varkappa) \, e^{-p_k |\tau|}, \quad (9)$$

we can manage to obtain an analytical solution of this problem. If we take the limit $\tau_0 \to \infty$ (i.e., $p_k \to 0$), neglect the heat conductivity, and assume that $F_{\chi_0}(\varkappa) = 0$, then the solution obtained for the fluctuations agrees very closely with the solution obtained in Ref. 16 for these conditions.

The relation (9) is chosen to simplify the calculations in deriving the above solution. A specific form of the τ-dependence is inessential, because the goal was to consider the situation when the time of coherence $\tau_0 = p_k^{-1}$ is much less than the time of nonlinear response of the medium. The derived solution has made it possible to determine the conditions under which the correlator splitting is possible in the equation for the coherence function. If we consider the nonlinear refraction parameter $R_T = L_d^2 / L_r^2$ ($L_d = ka^2$) and normalize the distance z to the length of thermal blooming L_r ($\bar{z} = z / L_r$), then this condition can be represented as

$$\sigma_{\chi_0}^2 \, \bar{z}^2 \, R_T \, \frac{\lambda_T \, \tau_0 \, r_k}{a^3 \, (1 + \lambda_T \, \tau_0 / r_k^2)} \ll 1. \qquad (10)$$

This condition limits, at the top, the values of R_T. However, for $\bar{z} \sim 1$ the values of the parameter R_T can be much higher than 1, under conditions that

$$\sigma_{\chi_0}^2 \ll 1; \quad \lambda_T \, \tau_0 \ll a^2; \quad r_k \ll a. \qquad (11)$$

These conditions need not be fulfilled simultaneously.

1.2. Radiation transfer equation in problems of thermal blooming

For the purpose of further simplifying of Eq. (2) we perform the following expansion into a Taylor series:

$$\Delta\varepsilon(z, \mathbf{R} + \rho/2) - \Delta\varepsilon^*(z, \mathbf{R} - \rho/2) =$$

$$= \varepsilon_r(z, \mathbf{R} + \rho/2) - \varepsilon_r(z, \mathbf{R} - \rho/2) +$$

$$+ i\left[\varepsilon_i(z, \mathbf{R} + \rho/2) + \varepsilon_i(z, \mathbf{R} - \rho/2)\right] \cong$$

$$\cong \rho\nabla_\mathbf{R} \, \varepsilon_r(z, \mathbf{R}) + 2i \, \varepsilon_i(z, \mathbf{R}) + i \, (\frac{\rho}{2} \nabla_\mathbf{R})^2 \, \varepsilon_i(z, \mathbf{R}). \quad (12)$$

Note that the expansion (12) is exact for the parabolic profile of the complex dielectric constant. Let us then drop the last term in Eq. (12), i.e., use the approximation

$$\Delta\varepsilon(z, \mathbf{R} + \rho/2) - \Delta\varepsilon^*(z, \mathbf{R} - \rho/2) \cong$$

$$\cong \rho\nabla_\mathbf{R} \, \varepsilon_r(z, \mathbf{R}) + 2i\varepsilon_i(z, \mathbf{R}) \,, \qquad (13)$$

and substituting (13) in (2) and then, performing the Fourier transform over ρ, we derive the equation

$$\frac{\partial J}{\partial z} + [\varkappa\nabla_\mathbf{R} + \frac{1}{2} \, \nabla_\mathbf{R} \, \varepsilon_r(z, \mathbf{R}) \, \nabla_\varkappa +$$

$$+ \alpha(z, \mathbf{R})] \, J(z, \mathbf{R}, \varkappa) = 0, \qquad (14)$$

where α is the absorption coefficient of the medium; $\alpha(z, \mathbf{R}) = k\, \varepsilon_i(z, \mathbf{R})$; J is the Fourier transform of the coherence function

$$J(z, \mathbf{R}, \varkappa) = (2\pi)^{-2} \iint\limits_{-\infty}^{\infty} \Gamma_2(z, \mathbf{R}, \rho) \times$$

$$\times \exp(-ik\varkappa\rho)\, d\rho. \qquad (15)$$

Note that Eq. (14) is equivalent to the radiation transfer equation (RTE) in a small-angle approximation, where J is the beam brightness or intensity. In quantum mechanics the function J is called the Wigner function. Strictly speaking, this name is more correct, because even if the function J satisfies Eq. (14), which is equivalent to RTE, but its characteristics do not fully correspond to the conception of brightness. In particular, the function J can be negative and differs from zero at points where the intensity vanishes.[27]

A remarkable feature of Eq. (14) is that it is of the first order in contrast to the initial equation for the coherence function Γ_2. The above procedures not only reduced the order of the equation but also the transfer was made from the complex function Γ_2 to real function J. Both of the cases are important when realizing the numerical algorithms of solving differential equations.

The radiation transfer equation has long been in use. It was derived first phenomenologically with the use of concepts of geometrical optics and has found a wide use in the classical theory of light transport. Later on the radiation transfer equation (RTE) was used with a success in the theory of neutron transfer. The relation between the RTE in a small-angle approximation and the equation for the second order coherence function obtained from the parabolic equation for the field was first determined in the Ref. 28 and then used in Refs. 27 and 29 to describe the radiation scattering in randomly inhomogeneous media. An essential fact that follows from these paper was determination of the electrodynamic meaning of the "radiation brightnessB concept. In Ref. 30 for statistically homogeneous media the RTE was derived from the equation of field coherence function in a more general case based not on the parabolic equation but on the Helmholtz equation. A more general case of statistically inhomogeneous media was considered in Ref. 31. As the studies on nonlinear atmospheric optics have been being developed, the scientists focused on the RTE. However, before the study described in Ref. 33 was published, different simplifications were used in solving the RTE based on the nonaberrational approximation, the phase screen approximation, or the method of moments.[16,32,34–38]

In Ref. 33 rigorous numerical solution of this equation for the axisymmetric case has first been carried out. The solution was obtained by the method of characteristics, which is traditional for the first order equations. These results have shown that it is impossible to generalize this algorithm to a five–dimensional problems.

It is essential to use additional approximations for solving such problems. As the above approximation, in Refs. 39 and 40, it has been first assumed that the asymptotic Laplacian method be used to calculate the integral representing general solution of the RTE. The use of such an approach enables one to turn from the numerical calculation of the set of equation characteristics, connecting a point in a present radiation plane with all the points of the initial plane, to calculation of a reference characteristic (a ray of the maximum brightness[39] or the geometrical optics ray[40]) or its first variations. Physically this means that the brightness distribution over angular coordinates at each point is approximated by a Gaussian distribution. This has made it possible to decrease the bulk of calculations by about two orders of magnitude. Using the methods of variation calculations and taking into account that the equations for the first variations are linear, a transition was realized in Ref. 40, with the use of Liouville formula, from setting the initial conditions to the given equations in the current plane to setting these in the initial plane. This resulted in a decrease by more than one order of the bulk of calculations needed. Later on other authors[41,42] have obtained, by making use of the idea of the Gaussian form conservation for brightness distribution, the solutions of the RTE. In Ref. 41, based on the square-law approximation for structure function of turbulent fluctuations of the air refractive index, an algorithm was realized, which enabled us to take into account the turbulent broadening of a partially coherent beam along with the nonlinear refraction.

1.3. Propagation of partially coherent radiation along the atmospheric paths

A combination of Laplacian method and the variation method has made it possible to construct very effective algorithms and to pass, for the first time, to solving five-dimensional problems. In these calculations we considered only one type of nonlinearity, namely, the thermal steady state (wind) nonlinearity. This is a consequence of the fact, that after intensive and not very successful seeking of effective modes for nonlinear propagation of high-power radiation along the atmospheric paths with different types of nonlinearity undertaken in the late 1970's, the scientists came to a conclusion that the way of avoiding the nonlinearity would be more effective. As a result we considered the beams of continuous (quasi-continuous) radiation with a large aperture, propagating in the upper rarefied atmospheric layers. In this case even for beams of 100-kW power the radiation density was 10^2 W/cm^2 that is at least two orders of magnitude below the threshold of manifestation of all the nonlinear effects except for the thermal one.

The calculations were made of the propagation of high-power beams along extended vertical paths for wide-band partially coherent radiation. In the

calculations we must consider that the wind direction varies with height above the ground.[43]   This causes strong distortions of the intensity distribution shape in the beam cross-section (Fig. 1).
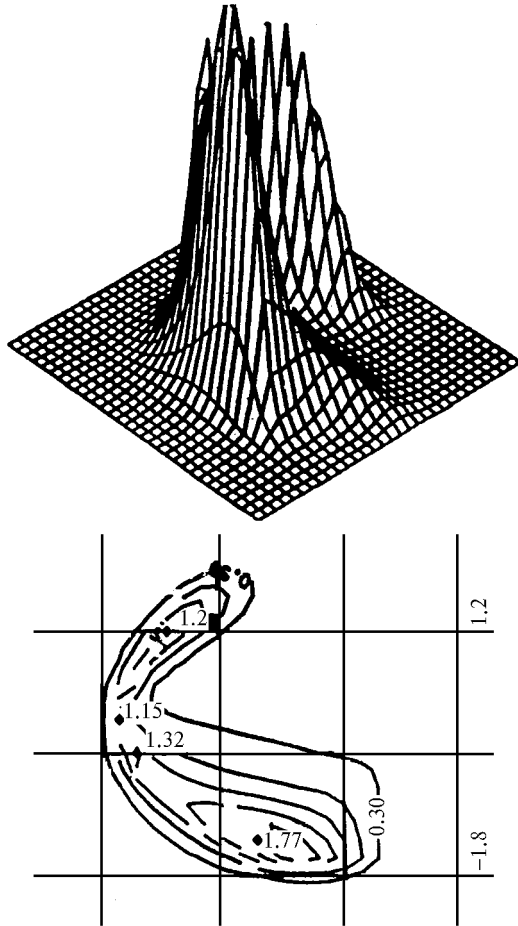

Fig. 1. Intensity distribution at 9 km altitude of the initially Gaussian beam.

1.4. Optimization of radiation propagation along the atmospheric paths. Optimization of the initial focusing

The primary goal of investigations into the radiation propagation along the atmospheric paths is to formulate recommendations on how to increase the efficiency of the radiation energy transfer. In the calculations on optimizing the partially coherent radiation focusing it was found that on vertical paths this focusing does not exhibit any peculiarities as compared with a coherent radiation.   For the paths with a nonlinear layer, adjacent to the source, the optimal focusing corresponds to the focusing at a point located in front of the receiving plane.[44,45]   A more precise focusing is required to compensate for the defocusing effect of this "nonlinear lens," and the region of the beam waist, while being important for homogeneous paths,[44–46] produces no effect because it lies outside the nonlinear layer of the medium.

For extended slightly slant paths the solution of the problem on the optimal focusing is not single-valued. These paths are inhomogeneous because along them the altitude above the ground varies. Maximum radiation absorption is observed at the path areas close to the surface. For different paths these areas can be at the beginning, at the end or in the middle of the path. The focusing conditions can be selected depending on the above concepts.

Now we consider the radiation propagation along the paths of 500 km long when the source is located at $H_0 = 10$ km and the receiver at $H_r = 15$ km.   This means that the angle γ of the path inclination relative to the horizon at the site of source location is $-1.7°$, i.e., the path is directed downwards.   Essential nonlinear beam distortions were observed for such extended paths at a relatively small radiation density (≈ 10 W/cm^2). The calculations made for three values of the angle of the initial radiation divergence, $\theta_d = a_0 / L_d$ (a_0 is the initial beam radius being equal to 1 m, L_d is the diffraction length of partially coherent radiation), $\theta_d = 2 \cdot 10^{-6}$, 10^{-6}, $2 \cdot 10^{-7}$ have shown that 14, 21, and 27% of power, reaching the receiving plane, fall within the receiving aperture, being a circle of 1-m diameter with its center at the original axis of the beam propagation.   The transmission of this path was rather high: $T = 94\%$ (only the molecular absorption was observed). For optimal distance of focusing the following values were obtained: $F_0 = 685, 750,$ and 745 km.

Note that for all the values of θ_d the radiation should be focused outside the receiving plane.   The same behavior of the optimal focusing is typical for homogeneous paths. However, as shown in Ref. 44–46, for homogeneous paths, F_0 monotonically increases with the decreasing θ_d value. This is because the nonlinear effects better manifest themselves on the homogeneous paths in the region of the focused beam waist. With a decrease in θ_d the power density in the waist increases, and the waist itself approaches the focal point. This path is not a homogeneous one. Its characteristic feature is that the effects of radiation absorption manifest themselves best of all at a distance of 185 km from the source where the path is most close to the Earth surface. In focusing outside the receiving plane, the beam power density is maximum at the end of the path or near its end (depending on the radiation power).

Noncoincidence of the region with maximum power density and the region of maximum radiation absorption results in the occurrence of two competitive regions of manifestation of nonlinear effects and, as a consequence, in the nonmonotonic dependence of F_0 on θ_d.

Similar results for optimization of the focusing were obtained for other paths with the initial angle of inclination $\gamma < -2°$. The calculations were made for the paths where the source and the receiver were located at altitudes from 10 to 25 km, and the path length was from 100 to 500 km. For the upward directed paths ($\gamma > 0$) another situation occurs, i.e., the optimal focal

V.E. Zuev et al.

Vol. 13, No. 1 /January 2000/ Atmos. Oceanic Opt.   31

length is less than the path length. This happens because on these paths nonlinear effects manifest themselves in the initial portion of the path.

A more precise focusing is required to compensate for the defocusing effect of this "nonlinear lens.B Such a situation is characteristic of inhomogeneous paths, with a nonlinear layer adjacent to the source.[44,45]

However, a simple conclusion that for the paths directed downward the radiation should be focused outside the receiving plane, and for the paths directed upward or horizontally, the focusing is required close to the receiving plane, follows from the calculations where the occurrence of the volcanic aerosol in the Junge layer in the stratosphere is not taken into account. This layer is at about 20 km altitude. As the calculations showed, when the beam propagated through the Junge layer, additional nonlinear distortions appeared due to absorption of radiation by aerosols. This essentially complicates the qualitative analysis of the situation, and to solve the problem on the choice of the initial focusing the numerical calculations must be made for each beam and each particular path.

1.5. Optimization of the radiation power

Among the first studies in numerical simulations of the propagation of coherent laser radiation along the atmospheric path was the study presented in Ref. 47.

The paper describes the results of investigations of the dependence of maximum intensity on the perturbation parameter

$$N_c = \left| \frac{dn}{dT} \right| \frac{\alpha \, I \, z^2}{\rho \, c_p \, v \, a_0} \qquad (16)$$

for different situations, namely, under conditions of beam scanning and without scanning, with the program phase correction and without it. It is shown that the parameters of the path and the initial focusing affect essentially the value and position of maxima of these dependences. When varying the value of the initial beam radius or the radiation wavelength, the slope of straight line changes, which corresponds to the propagation "without blurringB (i.e., diffraction propagation), and along with it the maxima positions change. Attempts of many researchers to determine the regularities of the dependence of the maxima positions on the parameter N_c based on theoretical analysis by inserting the correction factors taking into account focusing, scanning, beam scintillation, and so on,[48-50] did not improve the results since the accuracy of the above investigations was not high. Therefore the only way of solving this problem was the use of the numerical calculations of the laser beam propagation allowing for specific beam and path parameters. The optimization was made based on different criteria that did not alter the character of the behavior of the optimization curves, but affected the values of optimal power.[44,48]

For partially coherent radiation the situation becomes more complicated since one more parameter (coherence radius) occurs, on which radius the slope of a straight line depends, that corresponds to the beam propagation "without blurring,B and, hence, the shape of the optimization curves. However, based on the analysis of numerous calculations, made by the authors, we managed to determine stable characteristics of the optimal radiation propagation along different paths. Although this does not enable one to avoid numerical calculations, but provides for a drastic reduction of the required bulk of calculations.

In the Ref. 43 we managed, based on analysis of numerical calculations of high-power radiation propagation along a vertical path, to find the coordinate system where all the results fall on a straight line, and to establish that the effective solid angles of the beam outside the nonlinear layer at optimal propagation exceed the diffraction angles by a factor of 1.7 for winter atmospheric model and by a factor of 1.9 for the summer one.

From the above results it follows that if one manages to decrease the initial radiation beam divergence, then for the optimal propagation condition to be valid the initial power should be decreased by the same factor. In this case the effective power density increases by the same factor. A decrease in the initial divergence without an appropriate decrease of the initial power does not result in a marked increase of the effective intensity.

A criterion of optimal propagation, determined for vertical paths, turned out to be stable and manifested itself on slightly slant paths. The calculations were carried out for different paths, which differ both in length and altitude of location of sources and receivers. Different values of optimal power density were obtained. For the paths with the location of the source and the receiver at about 25-km altitude this power density exceeded by more than two orders of magnitude the corresponding values for the paths with the location of the source and the receiver at about 10 km altitude. However, in all the cases the conditions of optimal propagation corresponded to approximately the same nonlinear distortions. And in all the cases an effective beam size exceeded the diffraction-limited size of the beam by 30–50%, that is, as for vertical paths, in this case for an effective beam area the condition

$$R_{eff}^2 = (1.5 - 2.2) \, R_{dif}^2. \qquad (17)$$

should hold.

The value of the effective area increase can differ from two times, but this difference for most situations does not exceed 1.5 times. Thus in the numerical determination of optimal conditions for beam propagation we can use the case that if R_{eff} exceeds R_{dif} by less than 10%, then the value of the initial power is much less than the optimal one. And the excess of R_{eff} by more than two times as compared with

R_{dif} indicates that the initial power far exceeds the optimal one.

Another one important criterion of the optimal propagation can be represented by the condition

$$L_{r\ eff} \approx L,$$

where L is the distance; L_{reff} is the effective length of a nonlinear refraction, which is determined as

$$L_{r\ eff}^{-1} = \int_0^L (1 - z/L)\ L_r^{-2}(z)\ dz. \qquad (18)$$

The length of a nonlinear refraction L_r is determined by the following expression:

$$L_r[km] = 0.85 \sqrt{\frac{\pi\ a^3\ [m]\ v\ [m/s]}{P\ [kW]\ \alpha\ [m^{-1}]}},$$

which gives the value of the refraction length in km when substituting the values of the appropriate parameters expressed in the dimensions, shown in brackets. The type of this expression coincides with the traditional determination of the refraction length, but we substitute in this expression not the initial values of the beam radius (a), beam power (P), the values of transverse wind velocity (v), and the absorption coefficient (α) at the beginning of the path but their values varying along the path.

2. Remote diagnostics of the laser beam power and phase characteristics

2.1. Measurements of intensity and of its parameters' distribution

For measuring characteristics of the laser beam of high power and a considerable transverse size[51] the conventional techniques and devices used in the laboratory practice[52,53] turned out to be not always applicable. The use of matrices of receivers and bolometer networks[54–56] was limited by low mobility and difficulty in use on the slant and vertical paths. The necessity arose of the development of techniques of remote diagnostics of spatial and power parameters of the laser beams.

Owing to creation of infrared imagers[57] intended for remote measuring the spatiotemporal distribution of temperature of heated surfaces, the possibility appeared of reconstructing laser beam intensity distribution based on the temperature distribution over a surface heated with the beam. We have developed the thermophysical grounds for this technique of laser beam diagnostics.[58–61]

The problem of target heating by laser radiation formulated as the problem for solving three-dimensional equation of thermal conductivity when a heat flux on a heated surface is given as well as the conditions of heat insulation or cooling of back surface, is reduced using the method of invariant immersion[62,63] to the two-dimensional equations of thermal conductivity for the temperature of target surface with the equivalent heat sources corresponding to boundary conditions. Thus, for the heat regime of a "semilimited body" the corresponding equation of thermal conductivity has the form

$$\frac{\partial}{\partial t}\ T(\rho,\ t) - \lambda_T\ \Delta_\perp\ T(\rho,\ t) = -\frac{1}{8\sqrt{\pi^3}\ \lambda_T^{3/2}} \times$$
$$\times \int_0^t d\tau \int_{-\infty}^\infty d^2\rho'\ \frac{q(\rho',\ \tau)}{(t-\tau)^{5/2}} \exp\left(-\frac{(\rho-\rho')^2}{4\lambda_T(t-\tau)}\right) \quad (19)$$

for $T(\rho,\ t)$ which is the temperature on the target surface; $q(\rho,\ t)$ is the heat flux on the face surface connected with the intensity distribution over the beam cross section:

$$q(\rho,\ t) = (1 - R)\ I(\rho,\ t) + \vartheta T(\rho,\ t) - \sigma b T^4(\rho,\ t).$$

Here λ_T is the thermal diffusivity coefficient; ϑ is the heat transfer coefficient; b is the radiation coefficient; σ is the Stefan–Boltzmann constant; R is the reflection coefficient. Using Eq. (19) for the heat flux, we obtain

$$q(\rho,\ t) = -\frac{\lambda_q}{8\sqrt{\pi^3}\ \lambda_T^{3/2}} \int_0^t d\tau \int_{-\infty}^\infty d^2\rho'\ \frac{T(\rho',\ \tau)}{(t-\tau)} \times$$
$$\times \exp\left(-\frac{(\rho-\rho')^2}{4\lambda_T(t-\tau)}\right), \qquad (20)$$

where λ_q is the thermal conductivity coefficient.

Equations similar to (19), can be obtained for other thermophysical situations, important in practice, and the analytical relationships can be derived for reconstructing heat flux (and laser beam intensity distribution) according to the measured temperature distribution.[64] If the back surface of a heated target is a heat-protected, the analytical representation of heat flux on the target face surface is[60,61]:

$$q(\rho,\ t) = -\frac{\lambda_q}{4\pi\lambda_T L} \int_0^t d\tau \int_{-\infty}^\infty d^2\rho'\ \frac{T(\rho',\ \tau)}{(t-\tau)} \times$$
$$\times \exp\left[-\frac{(\rho-\rho')^2}{4\lambda_T(t-\tau)}\right]\frac{d}{d\tau}\theta_1\left(\frac{1}{2},\ \frac{\lambda_T}{L^2}(t-\tau)\right), \quad (21)$$

where L is the target thickness;

$$\theta_1(\vartheta,\ t) = 2\sum_{n=0}^\infty (-1)^n \times$$
$$\times \exp\left(-\pi^2(n+1/2)^2\ t\right) \sin \pi(2n+1)\vartheta.$$

For thermally thin targets[64] the equation for reconstructing the heat flux takes the form

$$q(\rho,\ t) = \frac{\lambda_q\ L}{\lambda_T}\left[\frac{\partial T}{\partial t}(\rho,\ t) - \lambda_T\ \Delta_\rho\ T(\rho,\ t)\right]. \quad (22)$$

If we have integral representations for reconstructing the heat flux, the relationships can easily be obtained to determine the laser beam integral parameters such as:

the radiation flux (laser beam power)

$$P(t) = \int\limits_{-\infty}^{\infty} \mathrm{d}^2\rho \, I(\rho, t), \qquad (23)$$

the vector of the center of gravity of the beam intensity distribution

$$R_c \{R_{c\,x}, R_{c\,y}\} = iR_{c\,x} + jR_{c\,y} =$$
$$= \frac{1}{P(t)} \int\limits_{-\infty}^{\infty} I(\rho, t) \, \rho \, \mathrm{d}^2\rho, \qquad (24)$$

the effective beam radius ρ_e determined by the relationship:

$$\rho_e^2 = \rho_{x\,e}^2 + \rho_{y\,e}^2 = \frac{1}{P(t)} \int\limits_{-\infty}^{\infty} I(\rho, t) \, \rho^2 \, \mathrm{d}^2\rho. \qquad (25)$$

In the case of, for example, a heat insulated target we have for a total flux the following expression:

$$P(t) = -\frac{\lambda_T}{L(1 - R)} \int\limits_{0}^{t} \mathrm{d}\tau \, M_0(\tau) \times$$
$$\times \frac{\mathrm{d}}{\mathrm{d}\tau} \, \theta_1 \left(\frac{1}{2}, \frac{\lambda_T}{L^2} (t - \tau) \right), \qquad (26)$$

where

$$M_0(t) = \int\limits_{-\infty}^{\infty} T(\rho, t) \, \mathrm{d}^2\rho.$$

The effective beam radius is

$$\rho_e^2 = -\frac{\lambda_q}{L(1 - R)P(t)} \int\limits_{0}^{t} \mathrm{d}\tau \times$$
$$\times [M_{T2}(\tau) + 4\chi(t - \tau) \, M_0(\tau)] \frac{\mathrm{d}}{\mathrm{d}\tau} \theta_1 \left[\frac{1}{2}, \frac{\lambda_T}{L^2} (t - \tau) \right],$$

where

$$M_{T2}(t) = \int\limits_{-\infty}^{\infty} T(\rho, t) \, \rho^2 \, \mathrm{d}^2\rho$$

is the moment of temperature inertia.

Because analytical relationships of the type (21) are the spatiotemporal convolution of the temperature distribution with multidimensional singular generalized functions,[65,66] the problem was solved on canonical regularization of the obtained functional expressions to develop numerical algorithms.[67]

The inverse problems of heat conductivity (IPHC) and the particular IPHC on recalculating boundary conditions, which is to be solved, refer to the class of ill-posed problems. The fact that it is ill-posed follows from the form of the obtained analytical relationships containing either differentiation of experimental data or a power singularity, that is equivalent to the differentiation, of a kernel of the integral relationship, which represents the solution of inverse problem. Therefore it is necessary to study stability of the problem on reconstructing the heat flux from data on temperature field with due regard of a random noise and to develop the reconstruction algorithms taking into account the measurement errors of the spatiotemporal distribution of temperature. Numerical simulation as well as the inversion of data of laboratory measurements make it possible to determine the accuracy and spatial resolution of algorithms constructed based on analytical solutions of the inverse problem.

Figure 2 shows the results of reconstructing heat flux for the case of a thin target (22),[68] which is a rectangular plate of $x_0 \times y_0$ size, and the temperature measurements are carried out at the time $0 < t < t_{\max}$.

A frequency-difference analog of equation (22) over spatial and temporal coordinate was used.[68] In this case we applied the filtration of temperature data by an optimal filter constructed on the basis of the Tikhonov smoothing functional.[69] An automated infrared imagery was used to measure the temperature distribution over the target surface.[70,71] Its parameters and the experimental conditions are described in Ref. 64. As the initial data we used 25 series frames of temperature distribution with the time interval 0.4 s with the spatial resolution 100×100 points. The result of the heat flux reconstruction from the measured values of temperature without noise suppression is shown in Fig. 2a.

Figure 2b shows the result of heat flux reconstruction with the use of a regularizing factor sinc(x). It is seen that in processing the image the artifacts, i.e., negative intensity values occur. The reconstruction of the heat flux using a regularizing algorithm (FFT) with the optimal Tikhonov filtration is shown in Fig. 2c. Figure 2 shows that the intensity distribution is close to a single-mode one. A comparison of the reconstructed and measured effective beam radii and focusing functionals (beam power is within the limits of a specified aperture) enables us to consider the results of reconstruction to be quite satisfactory ones.

Since the above method excludes the possibility of simultaneously obtaining data on laser radiation characteristics and the transient meters[54–56] available introduce additional distortions in the laser beam and do not provide measurements along slant paths, we have developed theoretical grounds for a tomographic method of reconstruction of the integral parameters of laser beams based on the brightness values of radiation scattered by a propagation medium.[72,73]

The problem of reconstruction is formulated as the problem of three-dimensional computational tomography, since using the transfer equation[76] one can determine[64] that the brightness of scattered radiation $J(\mathbf{r}, \Omega)$ and the laser beam intensity distribution $I(\mathbf{r})$ are connected by the integral Radon transform[77]:
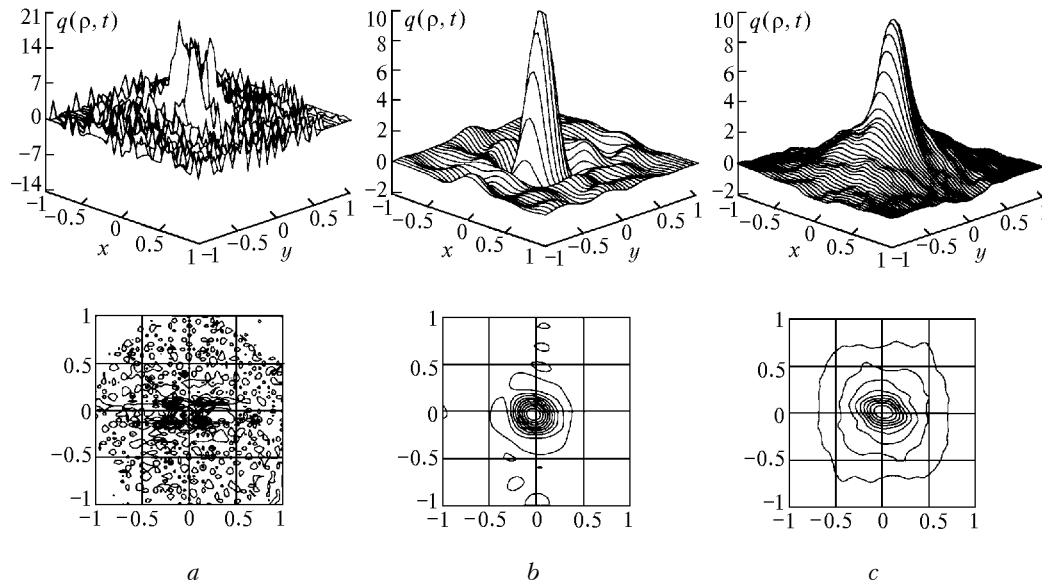
Fig. 2. Reconstruction of the heat flux, W/cm^2, is an example of processing data of a laboratory experiment; (*a*) heat flux reconstructed without filtration of measured temperature values; (*b*) heat flux reconstructed with the use of nonoptimal sinc-filter[64]; (*c*) heat flux reconstructed using the regularizing algorithm (FFT) with the optimal Tikhonov filtration.

$$J(\mathbf{r}, \mathbf{\Omega}) = (1/4\pi)\,\sigma_s(\mathbf{r}_0)\,f(\mathbf{r}_0, \varphi) \times$$

$$\times \exp\left[-\tau(\mathbf{r}_0, \mathbf{r})\right] \int\limits_0^\infty dR\, I(\mathbf{r} - R\mathbf{\Omega}), \qquad (27)$$

where $\mathbf{\Omega}$, $\mathbf{\Omega}_0$ are the unit vectors of the observation direction and the laser beam axis direction; $\sigma_s(\mathbf{r})$ is the volume scattering coefficient; $f(\mathbf{r}, \varphi)$ is the scattering phase function; $\varphi = \arccos(\mathbf{\Omega}\mathbf{\Omega}_0)$, $\mathbf{r}_0$ is the central coordinate of the region where the sight line and the laser beam intersect; $\tau(\mathbf{r}_0, \mathbf{r})$ is the optical thickness of the medium.

The estimations of the irradiance within the scattering volume image constructed with the receiving telescope, being a recorder of projection data for optical models of the Earth's atmosphere, including aerosol and molecular scattering of light, enable us to draw a conclusion about the prospects of reliable detection of the Radon projections of a laser beam from significantly high altitudes and about technical feasibility of the tomographic method in the Earth's atmosphere, including slant paths.

In Ref. 72 the calculating formulas and algorithms are given, which make it possible to determine the integral beam parameters omitting the stage of reconstruction of the intensity distribution. It is shown there that the reconstruction of the intensity distribution normalized to the total power and integral criteria of the beam quality can be established without the data on scattering and absorption characteristics of the medium. The numerical experiments showed that the reconstruction of integral criteria of the laser beam quality directly from the projection data does not lead to any loss of the reconstruction accuracy, as compared to the integration of the reconstructed intensity distribution, while, at the same time, makes the

reconstruction faster by more than one order of magnitude. Because we considered scattering of a laser beam by the Earth's surface in a linear approximation, i.e., without the account of the spatial modulation of its scattering properties by the radiation, the applicability of the developed algorithms is limited.

2.2. Determination of phase distortions

The development of coherent adaptive optical systems with the wave front control resulted in the formulation of the problem on wave front sensors − the problem of measuring phase distortions introduced by the atmosphere. The results of this problem solution have been generalized at certain stages.[44,78−81] However, the problem has arisen on evaluating the quality and improving efficiency of the current methods of wave front reconstruction. Moreover, the development of methods of phase reconstruction was far from being completed from the viewpoint of completeness of the data on measured wave front as well as of the possibilities of using for such a reconstruction the entire scope of phenomena connected with the diffraction and interference of laser beams.

Thus, for measuring phase distortions the instruments are used that allow one to estimate the wave front tilts. Among these instruments the Shak−Hartmann sensor is widely used. This device is a matrix of focusing elements whose apertures split the incident radiation being studied into partial beams.

The measured, with such a sensor, results must be transformed to the values of the phase itself. The available methods of such a transformation[82−84] do not allow effective estimation of the quality of wave front reconstruction to be made in the presence of noise as well as to select the subaperture shapes and the sensor position depending on the radiation geometry.
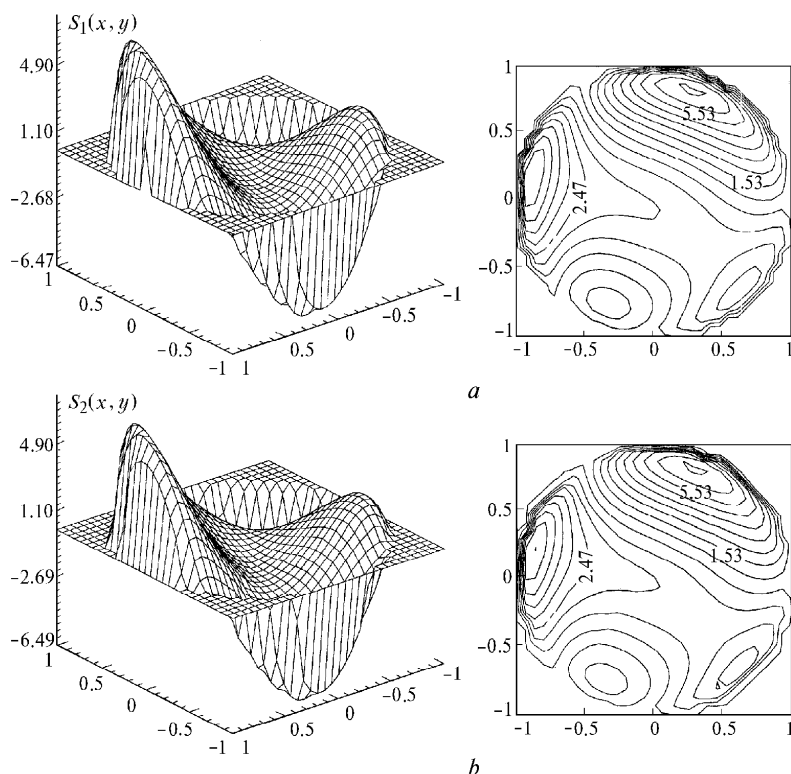
Fig. 3. Phase distribution within the entrance pupil: initial distribution (*a*) and reconstruded one using Eq. (28) (*b*).

It turned out to be quite a specific problem to measure the wave phase under conditions of strong atmospheric turbulence when the wave front dislocations can appear.[85,86] To overcome these difficulties we use an analytical representation connecting the phase with the value of its partial derivatives[64,87]:

$$S(x,y) = \frac{1}{2\pi}\int_C \frac{S(x',y')\ (y-y')\ \mathrm{d}x' - S(x',y')(x-x')\mathrm{d}y'}{(x'-x)^2+(y'-y)^2} +$$

$$+ \frac{1}{2\pi}\iint_D \mathrm{d}x'\mathrm{d}y'\ \frac{\mu(x',y')\ (x-x') + v(x',y')(y-y')}{(x'-x)^2+(y'-y)^2}\ . \quad (28)$$

Equation (28) expresses the phase in terms of the values of partial derivatives $\mu(x', y') = \frac{\partial}{\partial x} S(x, y)$, $v(x, y) = \frac{\partial}{\partial y} S(x, y)$ (wave front tilts) within the entrance pupil D and the phase value on the contour Γ limiting it.

This approach enables us to overcome the conditional separation into zonal and modal methods of phase measurements and representation in the systems of adaptive optics and to obtain the amplitude of modes regardless of the wave front approximation:

$$S(\mathbf{r}) = \sum_{k=0}^{\infty} b_k \Psi_k(\mathbf{r}), \quad (29)$$

where

$$b_k = \iint_D S(\mathbf{r})\ \Psi_k(\mathbf{r})\ \mathrm{d}^2\mathbf{r}\ , \quad (30)$$

that is performed by use of the basis functions $\Psi_k(\mathbf{r})$ be it Zernike polynomials or Walsh orthogonal modes, Haar functions, finite or boundary elements. In the numerical experiment on reconstruction of the phase and coefficients of its series expansion over a basis of Zernike polynomials the efficiency of the proposed approach was proved in constructing the control algorithms of wave front correctors for the adaptive optics systems.[88,89] The reconstructed function $S_2(x, y)$ is presented in Fig. 3*b*.

Such an approach was found to be also promising[90,91] in solving the problem of obtaining optimal, as averaged over an ensemble, the phase expansions over modes.

For such expansions the problem is formulated for minimization of an average over ensemble error energy $<\varepsilon^2>$, where

$$\varepsilon^2 = \iint_D \left[S(\mathbf{r}) - \sum_{k=0}^{N} b_k\ \Psi_k(\mathbf{r}) \right]^2 \mathrm{d}^2r, \quad (31)$$

and the basis $\Psi_k(\mathbf{r})$, using which this minimum is achieved, is called the Karunen−Loev−Obukhov basis. In Refs. 90 and 91 the possibility is demonstrated of representing the optimal mode expansions over any (set by a wave front corrector) system of functions.

In Ref. 92 we propose the principles of operation of a tomographic wave front sensor, in which the phase reconstruction is performed using measurement data on integral moments of the intensity distribution formed by the receiving objective in its focal plane at different positions of the diaphragm providing for tomographic scanning. The results of theoretical analysis and numerical simulation of this sensor have shown that at high precision of orientation of scanning and having measured position of the image center of gravity this method can not only offer major advantages of the Hartmann method over the interferometric one but also can provide for the precision of the latter.

As already mentioned, the reconstruction of the phase distribution of optical beams propagating in the atmosphere under conditions of strong turbulence, when the transverse intensity pattern clearly exhibits a speckle structure and when the wave front dislocations can occur, has become quite a special problem. The dislocations, decreasing essentially the efficiency of light energy transportation, are found to be "hiddenB for the wave front sensors of the available adaptive systems.[86]

A generalization of the semi-analytical approach[87] based on the potential and vortex properties of the vector field of phase gradient[92] enables one to make the dislocation (singular) phase distribution "visible.B

In this paper we have considered some aspects of a sophisticated complex problem on propagation of high-power laser beams through the atmosphere. The results of the investigations have been obtained at the Institute of Atmospheric Optics SB RAS in the past 15 years. Considerable attention has been given to the theoretical studies of the problems that could be of interest, in our opinion, to the readers, and which have not been sufficiently studied in the monographs published until so far.

References

1. L.A. Dyshko, Zh. Vychisl. Matem. i Matem. Fiz. **8**, No. 1, 238 (1968).
2. J.A. Fleck, J. Comp. Phys. **16**, No. 4, 324–341 (1974).
3. S.K. Godinov and V.S. Raben'ky, *Difference Schemes* (Nauka, Moscow, 1977), 439 pp.
4. I.J. Marchuk, *Methods of Computational Mathematics* (Nauka, Moscow, 1980), 535 pp.
5. V.V. Vorob'ev and V.V. Shemetov, "*Numerical investigation of some problems of thermal blooming of laser beams in the atmosphere*", Preprint, IFA AN SSSR, Moscow (1978).
6. K.D. Egorov and V.P. Kandidov, in: *Abstracts of Reports at Seventh All-Union Symposium on Wave Diffraction and Propagation*, Rostov-na-Donu (1977), Vol. 1, pp. 266–269.
7. Yu.N. Karamzin and A.P. Sukhorukov, "*Nonlinear interactions of diffracted light beam in media with square-law nonlinearity*, Preprint No. 43, IPM AN SSSR, Moscow (1974).
8. I. Wallace, J. Opt. Soc. Amer. **62**, 373 (1972).
9. K. Lantsosh, *Practical Methods of Applied Analysis* (Fizmatgiz, Moscow, 1961), 524 pp.
10. A.S. Roshal', Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **19**, No. 10, 1425–1454 (1976).
11. V.P. Kandidov, Usp. Fiz. Nauk **166**, No. 12, 1309–1338 (1996).
12. P.A. Konyaev, in: *Proc. of Sixth All-Union Symposium on Laser Radiation Propagation through the Atmosphere*, Part III, Tomsk (1981), pp. 195–198.
13. S.S. Chesnokov, Vestnik MGU, Fizika, Astronomiya **21**, No. 6, 27–31 (1980).
14. I.A. Fleck, I.R. Morris, and M.D. Feit, Appl. Phys. **10**, No. 2, 129–160 (1976).
15. V.A. Aleshkevich, G.D. Kozhoridze, and A.N. Matveev, Usp. Fiz. Nauk **161**, No. 9, 81–132 (1991).
16. V.V. Vorob'ev, *Thermal Blooming of Laser Radiation in the Atmosphere. Theory and Model Experiment* (Nauka, Moscow, 1987), 200 pp.
17. V.P. Kandidov and S.A. Shlenov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **27**, No. 9, 1158–1167 (1984).
18. V.A. Aleshkevich, S.S. Lebedev, and A.N. Matveev, Kvant. Elektron. **8**, No. 5, 1090–1094 (1981).
19. S.A. Akhmanov, Yu.E. D'yakov, and A.S. Chirkin, *Introduction to Statistical Radiophysics and Optics* (Nauka, Moscow, 1981), 640 pp.
20. V.E. Zuev, A.A. Zemlyanov, and Yu.D. Kopytin, *Nonlinear Optics of the Atmosphere. Modern Problems of Atmospheric Optics* (Gidrometeoizdat, Leningrad, 1989) Vol. 6, 256 pp.; A.A. Zemlyanov, Atmos. Oceanic Opt. **8**, No. 1–2, 44–57 (1995).
21. V.V. Dudorov and V.V. Kolosov, Kvant. Elektron. **28**, No. 2, 115–120 (1999).
22. G.A. Pasmanic, Zh. Eksp. Teor. Fiz. **66**, No. 2, 490–500 (1974).
23. A.S. Chirkin and F.M. Yusubov, Kvant. Elektron. **10**, No. 9, 1833–1842 (1983).
24. K.S. Gochelashvili, I.V. Chaskei, and V.I. Shishov, Kvant. Elektron. **7**, No. 10, 2077–2082 (1980).
25. B.S. Agrovsky, V.V. Vorob'ev, A.S. Gurvich, M.A. Kalistratova, D.P. Krindach, V.A. Myakinin, Kvant. Elektron. **7**, No. 1, 59–65 (1980).
26. V.V. Kolosov and M.F. Kuznetsov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **31**, No. 7, 816–822 (1988).
27. L.A. Apresyan and Yu.A. Kravtsov, *Radiation Transfer Theory* (Nauka, Moscow, 1983), 216 pp.
28. S.S. Dolin, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **7**, No. 3, 559–562 (1964).
29. L.S. Dolin, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **11**, No. 6, 840–849 (1968).
30. S.M. Rytov, Yu.A. Kravtsov, and V.I. Tatarskii, *Introduction to Statistical Radiophysics. Part II. Random Fields* (Nauka, Moscow, 1978), 463 pp.
31. Yu.N. Barabanenkov, A.G. Vinogradov, Yu.A. Kravtsov, and V.I. Tatarskii, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **15**, No. 12, 1852–1860 (1972).
32. V.V. Vorob'ev, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **13**, No. 7, 1053–1060 (1970).
33. V.V. Kolosov and A.V. Kuzikovskii, Kvant. Elektron. **8**, No. 3, 490–494 (1981).
34. P.Kh. Almaev, Tr. Ins. Exp. Meteorol., Akad. Nauk SSSR **B 18(71)**, 58–66 (1978).
35. M.S. Belen'kii and A.A. Zemlyanov, Kvant. Elektron. **6**, No. 4, 853–855 (1979).
36. M.S. Belkin, in: *Abstracts of Reports at Fifth All-Union Symposium on Laser Radiation Propagation through the Atmosphere*, Tomsk (1979), Part 3, pp. 27–31.
37. V.E. Zuev and Yu.D. Kopytin, Izv. Vyssh. Uchebn. Zaved., Ser. Fizika, No. 11, 79–105 (1977).
38. V.A. Petrishchev, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **14**, No. 9, 1416–1426 (1971).
39. A.A. Zemlyanov and S.N. Sinev, in: *Abstracts of Reports at VIII All-Union Symposium on Laser and Acoustic Sounding of the Atmosphere*, Tomsk (1984), Part 1, pp. 309–312.
40. V.V. Kolosov and M.F. Kuznetsov, *ibid*. pp. 327–330.

41. V.A. Banakh and I.N. Smalikho, Kvant. Elektron. **14**, No. 10, 2098–2107 (1987).

42. I.K. Babaev, M.S. Belkin, V.N. Koterov, A.G. Krasnovskii, and N.V. Cheburkin, Atm. Opt. **3**, No. 2, 119–124 (1990).

43. V.V. Kolosov, M.F. Kuznetsov, and S.I. Sysoev, Atm. Oceanic Opt. **5**, No. 4, 259–261 (1992).

44. V.P. Lukin, *Atmospheric Adaptive Optics* (Nauka, Novosibirsk, 1986), 248 pp.

45. P.A. Konyaev and V.P. Lukin, Izv. Vyssh. Uchebn. Zaved., Ser. Fizika, No. 2, 79–89 (1983).

46. S.A. Akhmanov, M.A. Vorontsov, V.P. Kandidov, A.P. Sukhorukov, and S.S. Chesnokov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofizika **23**, No. 1, 1–37 (1980).

47. L.I. Bradley and I. Herrmann, Appl. Opt. **13**, No. 2, 331–334 (1974).

48. D.K. Smith, Proc. IEEE **65**, No. 12, 59–103 (1977).

49. D. Strohbehn, ed., *Laser Beam Propagation through the Atmosphere* (Moscow, 1981), 416 pp.

50. F.G. Gebhardt, D.C. Smith, Appl. Opt. **11**, No. 2, 244–248 (1972).

51. B. Bunkin, V. Valuev, V. Glukhikh, G. Manukyan, V. Pismennyi, and A. Podin, Proc. SPIE **2025**, 29 (1993).

52. V.A. Zubov, *The Measurement Techniques of Laser Radiation Characteristics* (Nauka, Moscow, 1973), 192 pp.

53. A.F. Kotyuk, ed., *The Measurement of Energy Parameters and Laser Radiation Characteristics* (Radio i Svyaz', Moscow, 1981), 288 pp.

54. V.M. Kuzmichev, Yu.M. Latynin, and N.A. Priz, Prib. Tekh. Eksp. No. 2, 190–193 (1974).

55. V.V. Efremenko, Radiotekh. Elektron. **4**, No. 1, 193–196 (1979).

56. A.B. Katrich, A.V. Khudoshin, Avtometriya, No. 2, 108–110 (1987).

57. I. Sloid, *Systems of Infrared Imaging* [Russian translation] (Mir, Moscow, 1978), 414 pp.

58. V.P. Aksenov, E.V. Zakharova, and Yu.N. Isaev, Inzh. Fiz. Zh. **65**, No. 4, 622–628 (1995).

59. V.P. Aksenov, E.V. Zakharova, and Yu.N. Isaev, Atm. Opt. **4**, No. 2, 141–146 (1991).

60. V.P. Aksenov, E.V. Zakharova, and Yu.N. Isaev, Inzh. Fiz. Zh. **64**, No. 3–4, 275–280 (1994).

61. V.P. Aksenov and Yu.N. Isaev, Atm. Oceanic Opt. **5**, No. 5, 332–336 (1992).

62. J. Casti and R. Kalaba, *Embedding Methods in Applied Mathematics* (Aaldison-Wesley Publishing Company, Reading, 1973).

63. V.I. Klyatskin, *Method of Immersion in the Theory of Wave Propagation* (Moscow, Nauka, 1986), 256 pp.

64. V.P. Aksenov, V.A. Banakh, V.V. Valuev, V.E. Zuev, V.V. Morozov, I.N. Smalikho, and R.Sh. Tsvyk, *High-Power Laser Beams in Random-Inhomogeneous Atmosphere*, ed. by V.A. Banakh (Publishing House SB RAS, Novosibirsk, 1998), 341 pp.

65. I.M. Gelfand and G.E. Shilov, *Generalized Functions and Operations Using These Functions* (Gl. Izd. Fiz.-Mat. Lit., Moscow, 1959), 470 pp.

66. V.S. Vladimirov, *Generalized Functions in Mathematical Physics* (Nauka, Moscow, 1976), 320 pp.

67. V. Aksenov, Yu. Isaev, E. Zakharova, *Thermosense XVIII: International Conference on Thermal Sensing and Imaging Diagnostic Application*, Proc. SPIE **2766**, 336–356 (1996).

68. V.P. Aksenov, E.V. Zakharova, Yu.N. Isaev, A.V. Isakov, V.V. Reino, and R.Sh. Tsvyk, Atm. Oceanic Opt. **9**, No. 10, 859–862 (1996).

69. A.N. Tikhonov and V.Ya. Arsenin, *Methods for Solving Ill-Posed Problems* (Nauka, Moscow, 1979), 285 pp.

70. A.A. Dobotkin, A.V. Isakov, A.B. Il'in, A.P. Petrenko, V.V. Reino, R.Sh. Tsvyk, and M.V. Sherstobitov, Atm. Oceanic Opt. **7**, No. 5, 355–357 (1994).

71. A.V. Isakov, A.B. Il'in, A.P. Petrenko, V.V. Reino, R.Sh. Tsvyk, and M.V. Sherstobitov, in: *Abstracts of Reports at the XII Interrepublic Symposium on Laser Radiation Propagation through the Atmosphere and Water Media*, Tomsk (1993), p. 188.

72. V.P. Aksenov and V.V. Pikalov, Kvant. Elektron. **17**, No. 2, 167–172 (1990).

73. V.P. Aksenov and V.V. Pikalov, in: *Abstracts of Reports at the All-Union Workshop on Optical Tomography*, Tallin (1988), pp. 13–14.

74. D.S. Bochkov, V.A. Donchenko, and N.N. Latyshev, in: *Abstracts of Reports at the Seventh All-Union Symposium on Laser and Acoustic Sounding of the Atmosphere*, Tomsk (1982), Part 1, pp. 141–143.

75. G.G. Levin, E.G. Semenov, O.V. Starostenko, Opt. Spektrosk. **58**, No. 5, 1161–1164 (1985).

76. K. Keiz, Tsvaifel. *Linear Theory of Transfer* [Russian translation] (Mir, Moscow, 1972), 384 pp.

77. V.V. Pikalov and N.G. Preobrazhensky, *Reconstructive Tomography in Gas Dynamics and Plasma Physics* (Nauka, Novosibirsk, 1987), 231 pp.

78. M.A. Vorontsov and V.I. Shmalgauzen, *Principles of Adaptive Optics* (Nauka, Moscow, 1985), 336 pp.

79. M.A. Vorontsov, A.V. Koryabin, and V.I. Shmalqauzen, *Controlled Optical Systems* (Nauka, Moscow, 1988), 272 pp.

80. D.P. Luk'yanov, A.A. Kornienko, E.E. Rudnitsky, *Optical Adaptive Systems*, ed. by D.P. Luk'yanov (Radio i Svyaz', Moscow, 1989), 240 pp.

81. V.G. Taranenko and O.I. Shanin, *Adaptive Optics* (Radio i Svyaz', Moscow, 1990), 112 pp.

82. D. Fried, in: *Adaptive Optics* [Russian translation] (Mir, Moscow, 1980), pp. 332–348.

83. R.H. Hadgin, J. Opt. Soc. Amer. **67**, No. 3, 375–378 (1977).

84. A.N. Bogaturov, Izv. Vyssh. Uchebn. Zaved., Ser. Fizika **28**, No. 11, 86–95 (1985).

85. V.P. Aksenov, V.V. Kolosov, V.A. Tartakovsky, and B.V. Fortes, Atm. Oceanic Opt. **12**, No. 10, 912–918 (1999).

86. D.L. Fried, I.L. Vaughn, Appl. Opt. **31**, 2865–2882 (1992).

87. V.P. Aksenov and O.V. Tikhomirova, *Reconstruction of Optical Field Phase from the Wavefront Slopes*, Application of the Conversion Research Results for International Cooperation, Subconvers'99, Proc. IEEE **99EX246**, 30–32 (1999).

88. V.P. Aksenov and Yu.N. Isaev, Atm. Opt. **4**, No. 12, 166–172 (1991).

89. V.P. Aksenov and Yu.N. Isaev, Optics Letters **17**, No. 17, 1180–1182 (1992).

90. V.P. Aksenov and Yu.N. Isaev, Atm. Oceanic Opt. **7**, No. 7, 506–509 (1994).

91. V.P. Aksenov, V.A. Banakh, E.V. Zakharova, and Yu.N. Isaev, Atm. Oceanic Opt. **7**, No. 7, 510–512 (1994).

92. V.P. Aksenov and O.V. Tikhomirova, Proc. SPIE **3983**, 101–108 (1999).