USE OF LOCAL AND INTEGRAL CHARACTERISTICS OF HUMIDITY TO ESTIMATE ITS VERTICAL PROFILES

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Received July 1, 1996

We consider a possibility of reconstructing smoothed humidity profiles in the troposphere from the data on integral water vapor content and near-ground humidity value. Two techniques for reconstructing have been compared at the constant and variable index of the exponential profile. The errors in reconstructing, as well as the peculiarities of the humidity stratification over West Siberian region (city of Tomsk) and Canary Islands have been assessed from the results of comparison with radiosonde data. The variability of the vertical profile in different geophysical conditions have been considered.

INTRODUCTION

The data on vertical distribution of the atmospheric parameters are obtained by means of the expensive tools of aerological sounding (balloon-borne, airborne and radiosonde) or by lidar techniques. Other, more simple measurement methods make it possible to determine only the local characteristics of the near-ground layer, or their quantities integrated over entire atmospheric column, for example, light extinction coefficient and optical thickness of the atmosphere, absolute humidity in the near-ground layer and total water content, and so on. It is obvious from the general point of view, that if the vertical distribution of an atmospheric parameter may be described using a simple function, it is easy to reconstruct the profile itself from the integral and local data.

The results of experimental investigations obtained to date, and their generalizations in the atmospheric models[1–3] allow one to conclude that, at least in the troposphere, many parameters (humidity, aerosol number density, scattering coefficient, etc.) have an exponential dependence on altitude. The fact that the main part of the integral value of the parameter is determined or formed by the lower layers of the troposphere of a few kilometers in thickness is important for further consideration. These two circumstances are the basis for determining the smoothed vertical profile of the parameter studied, that in average corresponds to its actual distribution.

TECHNIQUE FOR RECONSTRUCTING THE HUMIDITY PROFILE

The relationship between the integral and local humidity characteristics and the vertical profile have been estimated in our previous paper.[4] Let us consider these relationships in application to real conditions based on the data of measurements of absolute humidity in the near-ground layer a_0 and the water vapor content in the atmosphere W_s obtained by spectroscopic method. Let us note that the formulas given below are valid for any another pair of local and integral atmospheric parameters (aerosol number density, scattering coefficient, etc.) which have the exponential vertical profile.

A. Let us first consider a simple case. Let us assume that the vertical distribution of humidity in the major part of the troposphere is described by the exponential function with constant exponent[10] β:

$$a(h) = a_0 \exp(-\beta h) \quad \text{at} \quad h < H , \tag{1}$$

and in the higher atmospheric layers the dependence, is either the same with a different exponent β or quite different. After integrating Eq. (1) over h, we obtain for the humidity height variation:

$$W(H) = (a_0/\beta)\,[1 - \exp(-\beta H)] . \tag{2}$$

Let us note that the value of the integral parameter $W(h)$ at the altitudes higher than several kilometers only weakly differs from its value integrated over the entire atmospheric thickness W_s. For example, according to the generalized data,[1] water vapor content of the 5-km layer is approximately 90% of the total water vapor content of the atmosphere. Taking into account this fact, after integrating Eq. (1) within the limits (0, ∞), we obtain the expression for β in the form

$$\beta = a_0/W_s . \tag{3}$$

Low content of the water vapor in high atmospheric layers and lower absolute variability give the grounds to consider different approximation for β. At the altitudes above the troposphere $h > H$ one can use the model values of the water vapor content $W_m(H, \infty)$, i.e., to calculate W_H by formula $W_H = W_s - W_m$. In this case one can obtain the following approximate formula from Eq. (2)

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$$\beta \approx \frac{a_0}{W_H}\left\{1 - \exp\left[\frac{a_0 H}{W_H}\right]\right\}. \quad (4)$$

The water vapor content W_m can be calculated, for example, based on the data of the mean zonal model.[1] Table I presents the model data W_m for two atmospheric conditions: tropics and midlatitude summer. We took the altitude $H = 8$ km as the upper boundary, though it is not principle, because close results are obtained if other altitudes $H = 6$ to 10 km are used.

TABLE I. Estimate of the water vapor content of the atmosphere from the data of mean zonal model.[1]

Characteristics	Midlatitude summer	Tropics
W_m(0–60 km), g/cm^2	2.65	3.64
W_m(8–60 km), g/cm^2	0.14	0.24
a_0, g/m^3	11.97	17.71

Model estimates of the accuracy of approximations (3) and (4) showed that the deviations from the actual values of the index β do not exceed 2%. Non exponential distribution of humidity in the near ground layer and presence of inverse layers can essentially affect the error in reconstructing $a(h)$. If the information on the state of the near-ground layer is available, the technique under consideration may be improved, but we do not consider such a variant in this paper.

B. In a more sophisticated presentation of the dependence $a(h)$, let us consider the fact known from the experimental data that the tropospheric profile is described more exactly by the exponent with the exponent β variable with the altitude.[1,5] Concretely, the decrease in humidity with altitude increase, and the current value β can be written, for example, using the weighting coefficient

$$\beta(h) = \beta_1\left[(H - h)/H\right] + \beta_2'\,(h/H). \quad (5)$$

Then the altitude dependence $a(h)$ takes the form

$$a(h) = a_0 \exp\left[-\beta_1 h - \beta_2 h^2\right], \quad (6)$$

where $\beta_2 = [(\beta_2' - \beta_1)/H]$, $\beta_2 \ll \beta_1$. Estimates of the value β_2 showed that it is equal to 0.0192 km^{-2} for the average conditions (midlatitude summer[1]) and coincides with the value of analogous parameter in the Zuring formula.[5]

To determine the parameter β_1, let us integrate Eq. (6) over the altitude from 0 to ∞. The exact solution of the integral for the water vapor content W_s is expressed in this case by the probability integral[7]

$$W_s = a_0\sqrt{\frac{\pi}{2}}\left\{\frac{\exp\,(\beta_1^2\; 4\beta_2)}{\sqrt{2\beta_2}}\left[1 - \mathrm{F}\left(\frac{\beta_1}{\sqrt{2\beta_2}}\right)\right]\right\} =$$

$$= a_0\sqrt{\frac{\pi}{2}}\; Y(\beta_1,\beta_2). \quad (7)$$

The solution of Eq. (7) relative to β_1 does not reduce to the algebraic equation, so we used an approximation. The values of the function $Y(\beta_1, \beta_2)$ were first calculated for the expected values β_1 at the set β_2. Then an approximate expression was selected for the obtained dependence (Fig. 1) in the form of the function

$$\beta_1 \cong 0.7961\; Y^{-1} - 0.04428\; Y, \quad (8)$$

where $Y = W_s / a_0 \sqrt{0.5\text{p}}$. Let us note that the error of the approximation (8) does not exceed 1% at the mean value of 0.1%. Thus, the final expression for calculating $a(h)$ is of the form

$$a(h) = a_0 \exp\left\{-\frac{1.25 a_0\, h}{W} + \frac{0.0353 W\, h}{a_0} - 0.0192\, h^2\right\}. \quad (9)$$

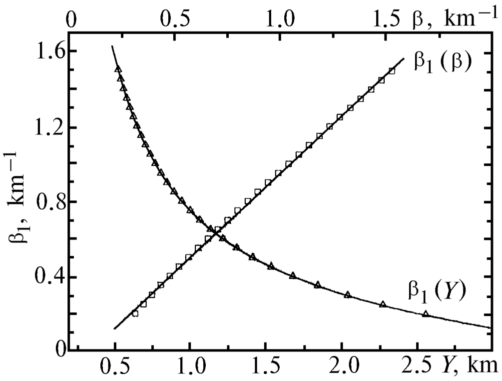

FIG. 1. Illustration of the dependences $\beta_1(Y)$ *and* $\beta_1(\beta)$.

Similarly to a more simple variant (see A), the solution for β_1 can be expressed by the water vapor content within a limited height range W_H (is not presented because it is too cumbersome). The analysis of this case showed that the complication of the solution related to the integration within the limits $(0, H)$ practically does not improve the accuracy the parameters β determination.

COMPARISON WITH THE RADIOSONDE DATA AND MODELS

To assess the applicability of the technique to reconstruction of $a(h)$, the data of simultaneous measurements of water vapor content W_s and radiosonde sounding data on $a^{\mathrm{r}}(h)$ carried out near the city of Tomsk[4,8] in 1992 and 1995 and near Canary Islands[9] in 1994 were analyzed. The series of results of radiosonde and reconstructed profiles $a^{\mathrm{rec}}(h)$ are shown in Fig. 2 and 3 for illustration. The model profiles[1,6] $\overline{a^{\mathrm{m}}}(h)$ are shown here for a comparison, as well as the profiles $a^{\mathrm{m}}(h)$ obtained using the same model stratification but related to the actual humidity a_0 in the near-ground layer.

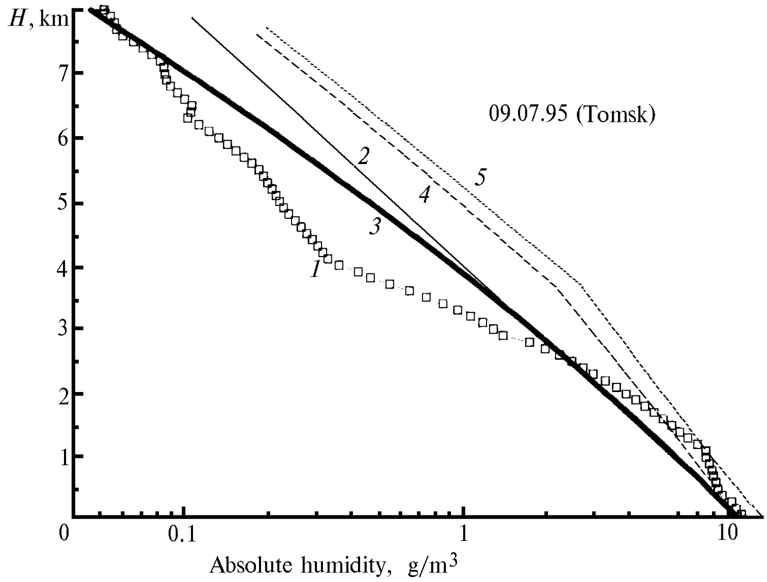

FIG. 2. Illustration of the humidity profile over the city of Tomsk from the radiosonde data (curve 1) reconstructed by the techniques A (curve 2) and B (curve 3), and by the mean-zonal model taking into account near-ground humidity (curve 4) and without that (curve 5).

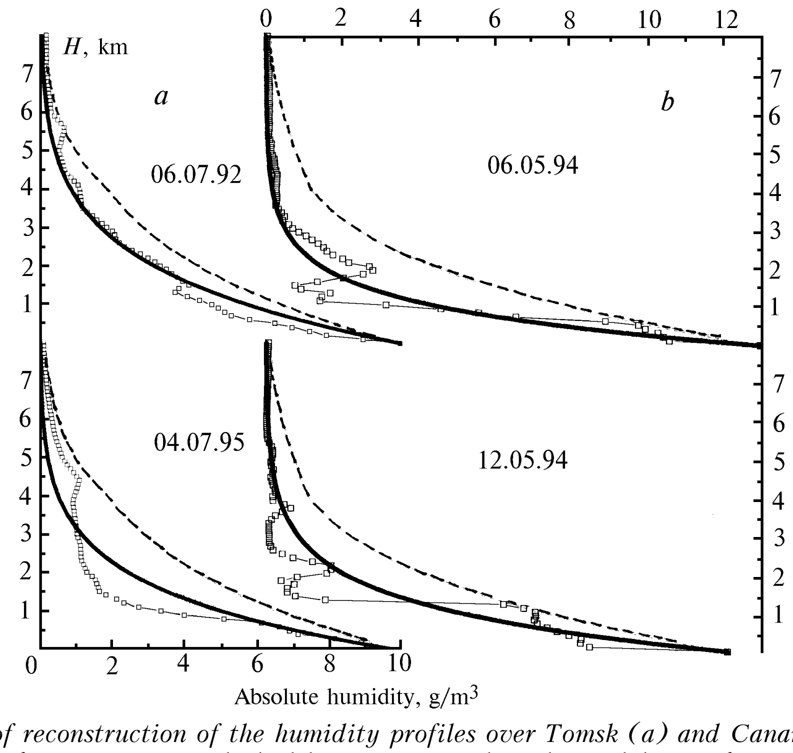

FIG. 3. Examples of reconstruction of the humidity profiles over Tomsk (a) and Canary Islands (b). Solid lines are the results of reconstruction, dashed lines correspond to the model stratification taking into account near-ground humidity, squares are the radiosonde data.

The results of comparison of different profiles make it possible to draw some obvious conclusions. The maximum difference with the actual values $a^{\mathrm{r}}(h)$ is observed for the mean-zonal model $\overline{a^{\mathrm{m}}}(h)$, because by its definition, it reflects only the mean humidity distribution in the geographical zone considered. Profiles $a^m(h)$ are quite close to $a^r(h)$ in many cases of measurements over Tomsk, due to their relation to the actual humidity in the near-ground layer a_0. Profiles $a^{rec}(h, \beta)$, on the average, well describe the smoothed behavior of the humidity up to 4–5 km, but often give higher values at the top of the troposphere. At last, the dependences $a^{rec}(h; \beta_1, \beta_2)$ describe the mean humidity distribution in the entire altitude range most correctly.

Common peculiarities of the profiles obtained in the Canary Islands region are great gradient of humidity and great inversions in the near-ground layer, that finally leads to the great deviations from the model conception about the increase of β with the increase of altitude. In this connection, the simpler technique A was used.

To quantitatively estimate the error in reconstruction of the humidity stratification, the absolute, δ, and relative, ε, errors were calculated

$$\delta = |\overline{\Delta}| + S_\Delta\,, \quad \varepsilon = (\delta/\overline{a^r}\,)\,100\%\,, \qquad (10)$$

where $\Delta_i = (a^r - a^{rec(m)})$, $\overline{\Delta} = \sum_{i=1}^{n} \Delta_i/n$,

$\overline{a^r} = \sum_{i=1}^{n} a^r/n$ is the number of measurements a^r in an altitude interval, and the rms error of the difference between the radiosonde data and calculated values $a^{rec}(a^m)$ was calculated by the formula

$$S_\Delta = \sqrt{\sum_{i=1}^{n} (\Delta_i - \overline{\Delta})^2/(n-1)}\,.$$

All data of radiosonde observations, i.e. 9 profiles over Tomsk and 12 profiles of marine measurements were used for calculations.

One can select three intervals in the altitude dependence of the data obtained in Tomsk (Fig. 4*a*). Absolute errors of different techniques are maximum and comparable in the near-ground layer up to 2 km, because they are mainly caused by non-exponential dependence of $a(h)$. The errors in local-integral reconstructing for the altitude interval 2 to 5 km are approximately 2 times less than the model ones, and the decrease of the error with altitude is observed. The second technique $a(\beta_1, \beta_2)$ gives good results in the upper part of the troposphere, and the error of the profiles $a(\beta)$ increases. On the whole, the relative error in reconstructing $a(\beta_1, \beta_2)$ is from 10 to 60%.

The advantage of using the local-integral technique for reconstructing is more obvious for estimation of the errors in the marine region (Fig. 4*b*). For example, comparison of the relative errors ε shows that they are 3–4 times greater for the model profiles (except for the altitudes 7–8 km). The errors are much greater here for any technique of reconstruction including the model one than in the

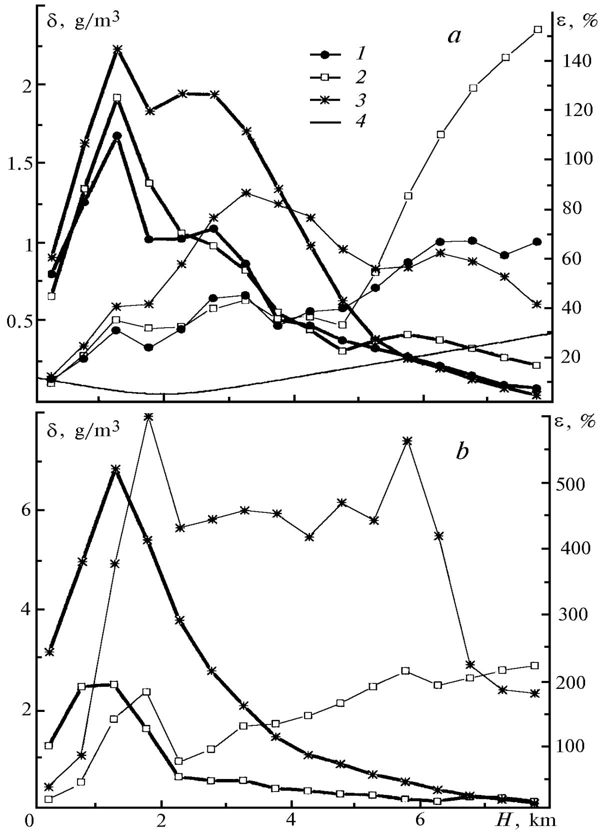

FIG. 4. Estimate of the altitude dependence of the absolute, δ, (solid lines) and relative, ε, (thin lines) errors in reconstructing a(h) under the conditions of Tomsk (a) and Canary Islands (b): 1) $a^{rec}(h, \beta_1, \beta_2)$, *2)* $a^{rec}(h; \beta)$, *3)* a^m, and *4)* $\Delta a^{rec}/a^{rec}$.

midlatitude continental region (Tomsk). The main reason for the aforementioned fact is the specific peculiarity of stratification $a(h)$, a sharp decrease of humidity in the narrow altitude range up to 1.5 km to extremely low values. Humidity actually restores its typical values only at the altitudes of 7–6 km (it is the reason, why the error of model profiles decreases at these altitudes). The consequence of extremely low humidity at the altitudes 1.5 to 7 km is their small effect on the measured water vapor content, i.e., W_s becomes not sensitive to the humidity distribution.

The results of estimation of the effect of instrumental errors in measuring W_s and a_0 are also shown in Fig. 4*a* (curve *4*). The values $\Delta W_s/W_s \le 4\%$ and $\Delta a_0/a_0 = 10\%$ are used as the errors.[8,13] The results of estimation of the effect of variations of the measured characteristics W_s and a_0 on the estimate of $a(h)$ give the qualitatively similar vertical behavior even at zero measurement error. Thus, there is an obvious conclusion that the use of additional information about W_s does not improve the estimation $a(h)$ at the altitudes above 6–8 km in comparison with the models.

VARIABILITY OF THE EXPONENT β

As follows from the analysis performed, β is a simple and unambiguous parameter that characterizes the smoothed vertical distribution of humidity in the troposphere. There are no direct data on the variability of the parameter β found in literature so far. One can obtain some indirect estimates only from the analysis of the results of modeling[1] or from the fact of a relatively high correlation between W and the near-ground humidity[4,10] (hence, the small variability of β). From this point of view, it was interesting to consider statistical characteristics of short-period variations of the exponent in the formula for vertical profiles.

The results of calculations of β and β_1 (the parameters in the techniques A and B, respectively) show that they are related practically linearly. For example, the approximate relationship for the midlatitude summer conditions has the form (see Fig. 1)

$$\beta_1 = 1.06\,\beta - 0.104\,. \tag{11}$$

So the statistical estimates have been made only for one, more simple for calculations, parameter β (Table II). It follows from the data presented that the difference between the spring and the summer values β is about 20% for the region near Tomsk. Summer profiles have, on the average, greater humidity gradient. Average value of the exponent β near Canary Islands is two times greater than the corresponding value for midlatitudes. As compared to the mean subtropical conditions, the profiles $a(h)$ in the Canary Islands region with anomalously great β characterize the humidity deficit of the middle troposphere. Such a behavior of $a(h)$ manifests itself in the altitude dependence of the exponent β. The analysis of actual profiles shows that, deviations from normal conditions, i.e., the decrease of β with altitude is observed in many cases at $h \approx 4$ to 6 km. Obviously, the stratification considered is caused by the peculiarities on circulation processes during the period of observations, i.e., the primary emissions of dry air from the deserts of western Africa. Let us pay attention to the fact that the water content of subtropical region near African shore was two times lower than that in midlatitudes at close values of near-ground humidity (see data arrays 2 and 3).

TABLE II. Statistics of the β, W *and* a_0.

No.	Region, season		Mean	rms	V	min	max	N
1	Tomsk, Spring 1992–1993	β	0.443	0.146	0.33	0.181	0.846	44
		W	1.0	0.35	0.35	0.38	1.92	
		a_0	4.22	1.43	0.34	0.81	7.39	
2	Tomsk, Summer 1992–1995	β	0.539 (0.568)	0.121 (0.145)	0.22 (0.260)	0.325	0.837	(326) 56
		W	2.42	0.63	0.26	1.25	3.76	
		a_0	12.59	2.80	0.22	6.95	18.15	
3	Canary Islands, May 1994	β	1.05	0.20	0.19	0.761	1.484	14
		W	1.20	0.24	0.20	0.802	1.786	
		a_0	12.2	1.57	0.13	9.22	15.13	

The relative day-to-day variations of β (variation coefficients V_β) are approximately the same in summer over the continent and in the marine region (about 20%) and are comparable with the variations of a_0 and W. The values V_β are a little bit greater in the transitional period (spring), but remain at the level of relative variations of humidity a_0 and water vapor content.

It follows from the statistics of hourly average values of β (the data are presented in parentheses in Table II) that the absolute (rms) and relative (V) variations are insignificantly different than those of day-to-day variations. One should suppose on this basis that the oscillations of synoptic and higher scales play a principal role here. Nevertheless, the analysis of diurnal behavior have revealed some regularities (Fig. 5). The morning maximum in diurnal behavior of the water vapor content in summer 1992–1995 (in contrast to summer and spring data of 1992–1994) is only weakly pronounced, and mainly the increase of W by 4–6 p.m. of local time is observed. The well known regularity of diurnal behavior of near-ground humidity[5] a_0 (daytime maxima at 10 a.m. and 4 p.m.) is not so pronounced. The result of such a variability of W and a_0 is a more pronounced diurnal behavior of $\beta(t)$. Evaporation and increase of $a(h)$ in the lower tropospheric layers begin in the morning and, as a result, β increases by noon. Then the convection and turbulence make water vapor content uniform what results in a decrease of β. Next increase in β occurs in the evening as the turbulence becomes weaker. The mean total amplitude of diurnal variations is 10–12%, according to our data.

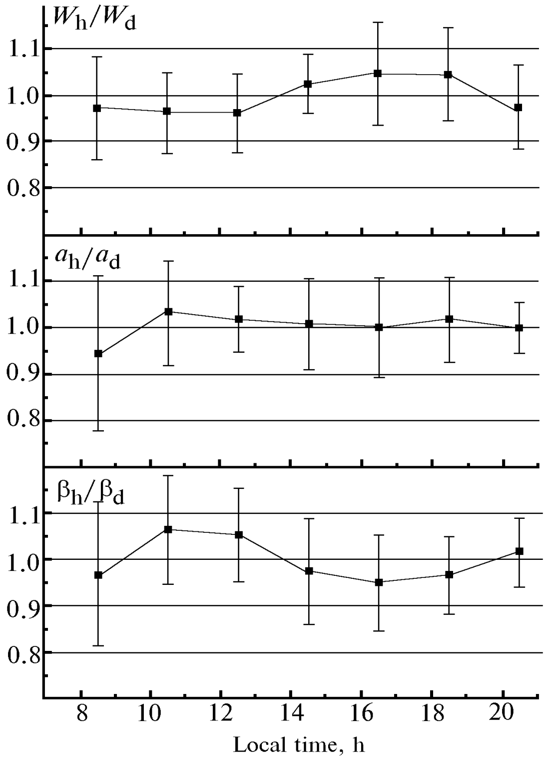

FIG. 5. Diurnal behavior of the normalized humidity parameters during the summer period of observations in Tomsk in 1992–1995. Subscripts at the values are "h"-hours, and "d"-days).

Simple relation of the vertical profile exponent to the humidity parameters (Eq. (3)) allows us to estimate the range of seasonal variability of β from the data obtained at the meteorological network. According to the long-term monthly mean data for the city of Tomsk,[11,12] the relative annual amplitude (ratio of the maximum monthly mean value to the minimum one) of the near-ground humidity is one order of magnitude, water content amplitude is 6.9, and the exponent β amplitude is only 1.7. Another important peculiarity is that, in contrast to the smooth annual behavior of W and a_0, monthly mean values β can be certainly divided into two groups, within which their variations are not significant (Fig. 6). The mean value of β is 0.31 from November till March, and then it suddenly changes to the "summer" values in April, and remains practically constant at the level of 0.43 until October. That subdivision into two periods is in a good agreement with the time when the average temperature passes through 0°C point. In the region of Tomsk this happens in April and October. Thus, the month-to-month variation of β is principally related to the phase transition from snow covered surface to the moistened one, and, correspondingly, to different conditions of evaporation and convection of the water vapor.

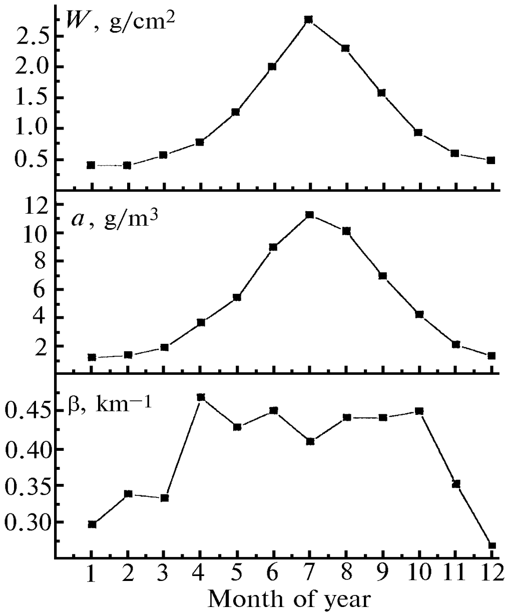

FIG. 6.

The difference between the presented estimates of β and our summer data (β during warm period is about ~ 0.5, see Table II) can be explained by two reasons. The daily mean values of a_0 and W are the basis for the estimation of the long-term monthly mean β, but only daytime observations are used in Table II. The second reason is connected with different ways of averaging when estimating β: the ratio of the mean values $\overline{a}_0$ and $\overline{W}$, or the mean value of the ratio a_0/W, respectively.

The results obtained allows one to draw the following conclusions

1. The local-integral technique proposed for reconstructing separate smoothed out humidity profiles in the main altitude range (up to 5–6 km) is characterized by the mean error of about 30% under typical conditions (city of Tomsk) and about 100% under extreme conditions (Canary Islands). The error of reconstruction in both cases is 2–4 times less than that for model representation.

2. The maximum absolute errors in reconstruction of humidity are characteristic of the near-ground layer (δ = 1...1.7 g/m^3 for Tomsk). Further improvement in the local-integral technique at these altitudes is possible if one takes into account the peculiarities in stratification some tropospheric layers and the specific features of the distribution over some geographical regions, i.e. using certain a priori information.

3. Total range of variability of the exponent in the vertical profile function β is 0.18 to 0.84 during warm period in Tomsk, the mean value in spring is 0.44 and 0.54 in summer. The value of β on Canary Islands are very high, what is indicative of the large vertical gradient and humidity deficit in the middle part of the troposphere.

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